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Tangent Lines

Unit 12 Lesson 1

Students will be able to:

Understand the theorem of tangent lines in a circle, prove and solve problems involving tangent of a circle.

Key Vocabulary:

- Circle
- Radius
- Tangent Lines
- Point of Tangency
- Pythagorean Theorem
- Right Triangle
- Right Angle

REVIEW BASIC TERMS

CIRCLE - is the set of all points in a plane that are equidistant from a given point in the plane called the center.

RADIUS- is a segment with one end point at the center and the other end point on the circle.

CHORD - is a segment that joins two points on the circle.

DIAMETER- is a chord through the center of the circle.

SECANT- is a line contains a chord.

TANGENT- is a line in the plane of the circle that intersects the circle in exactly one point is a tangent to the circle.

POINT OF TANGENCY- is the point where the tangent line intersects the circle.

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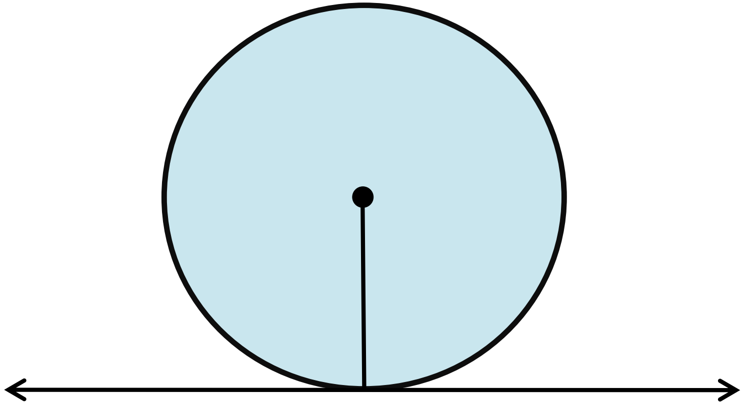
TANGENT LINES

THEOREM 1: If a line is tangent to a circle, then it is perpendicular to the radius at its outer endpoint.



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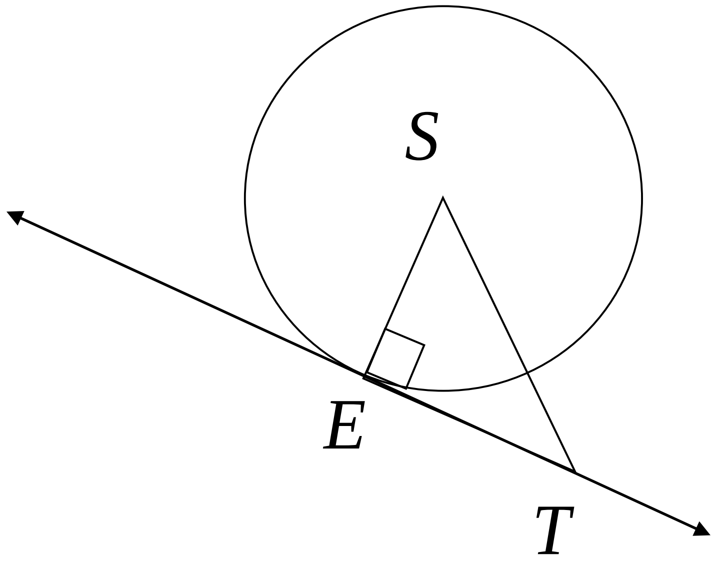
Sample Problem 1

1. In the figure, SE is a radius and ET is the tangent to the circle at E. If SE = 12 cm and ET = 16 cm, how far is it T from the center S?



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Sample Problem 1

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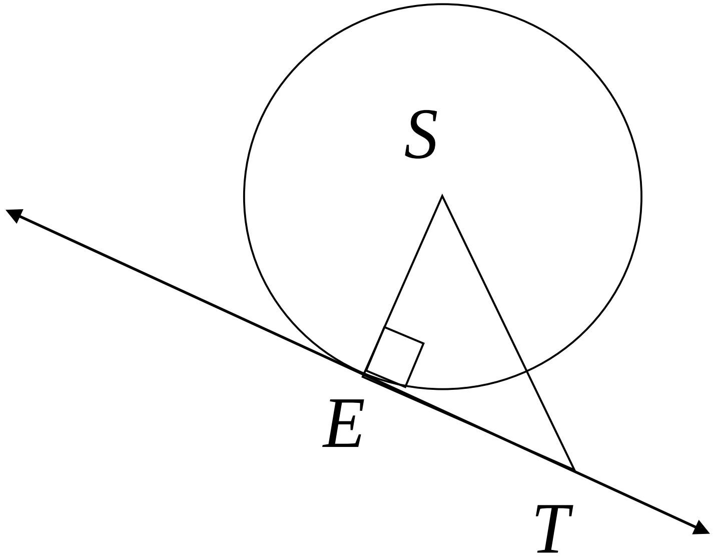

Solution:

$$ST = \sqrt{SE^2 + ET^2}$$

$$= \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400}$$

$$= 20cm$$

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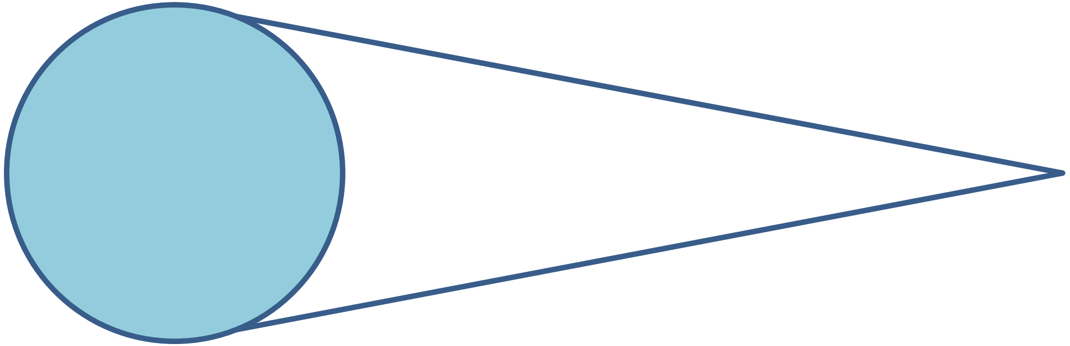
Theorem 2: If two tangent segments are drawn to a circle from an external point, then

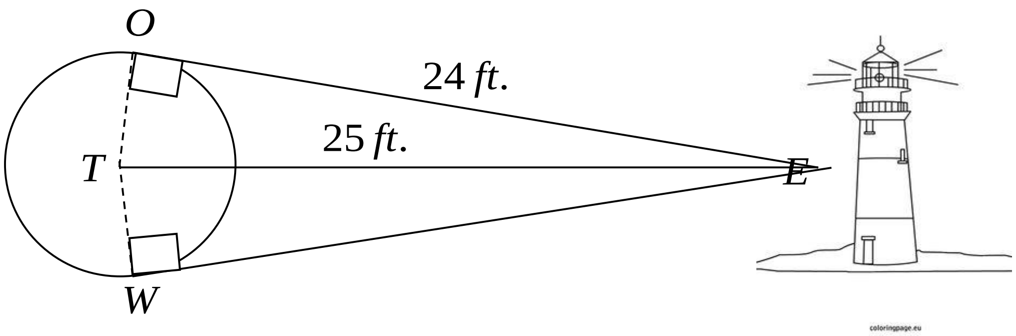
A. The two segments are congruent, and

B. The angles between the segment and the line joining the external point to the center of the circle are congruent.



Sample Problem 2:

The figure shows the distance from the base of the light house to a circular garden. What is the radius of the garden?



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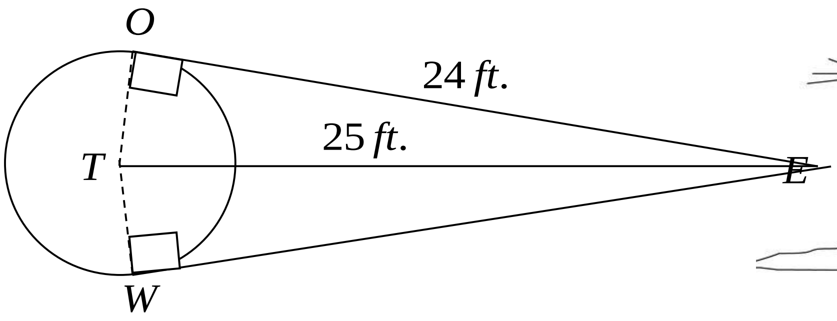
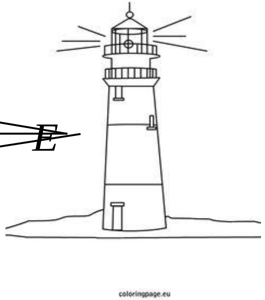




Solution:

$$(TO)^2 + (OE)^2 = (TE)^2$$

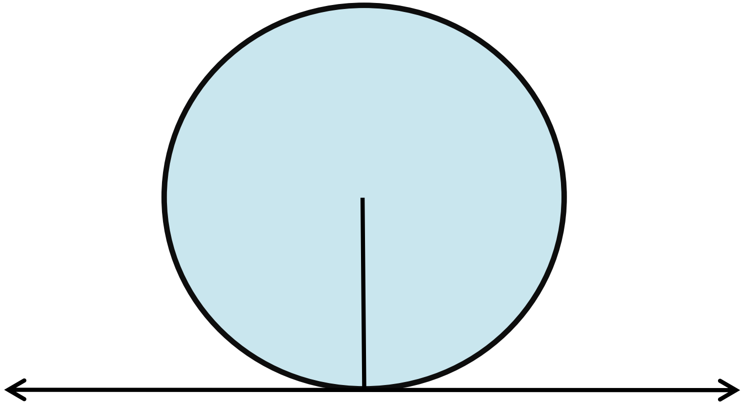
$$r^2 + (24)^2 = (25)^2$$

$$r^2 + 576 = 625$$

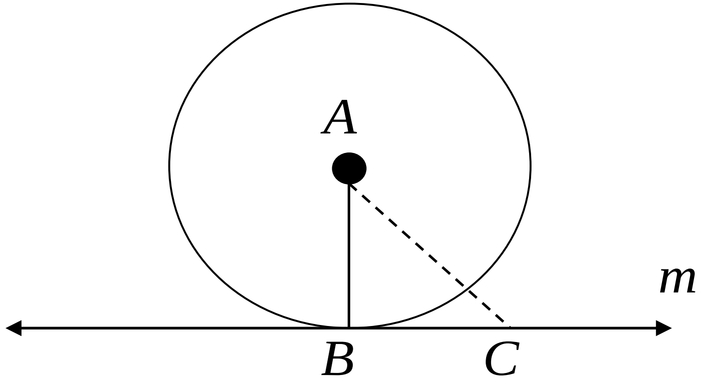
$$r^2 = 49$$

$$r = 7$$

THEOREM 3: If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle.



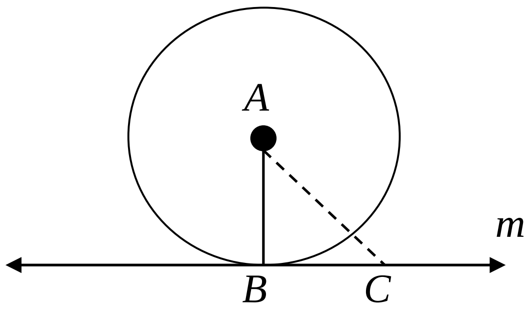
Example:



Given: ⊙A with *AB* a radius and *AB* ⊥ *m* at point B

Prove: Line *m* is a tangent to ⊙*A* at *B*.

Example:



Given: ⊙A with *AB* a radius and *AB* ⊥ *m* at point B

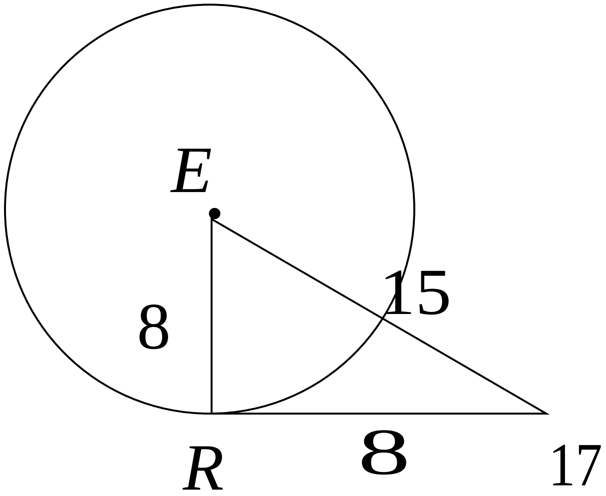
Prove: Line *m* is a tangent to ⊙*A* at *B*.

Proof:

Statement	Reasons
Let C be any point on m distinct	A line maybe named by any of its two points.
Draw AC.	The line postulate
AB ⊥ m at B.	Given
∠ABC is a right angle.	Definition of perpendicularity
ABC is a right triangle	Definition right triangle
AC > AB	The first Minimum Theorem
C is exterior of ⊙A at B.	C is any point except B. If a line touches a circle at only one point, then it is a tangent to the circle.

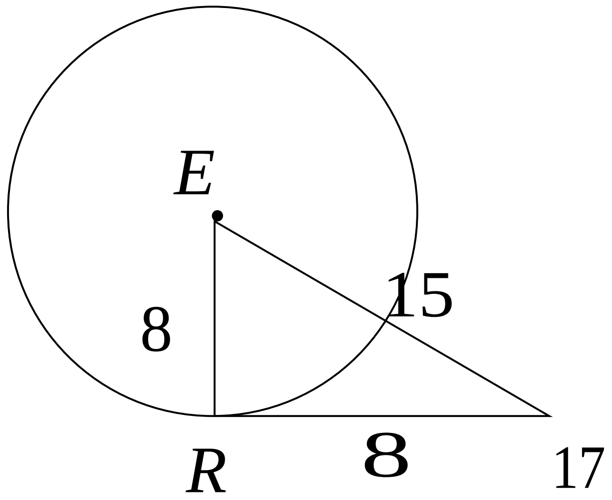
Sample Problem 3:

3. Determine whether ER is tangent to ⊙ P at E.



Sample Problem 3:

3. Determine whether ER is tangent to ⊙ P at E.



Solution:

If △ PRE is not a right triangle, then ER is not a tangent to P at E. Apply the converse of Pythagorean Theorem to check whether △ PRE is a right triangle.

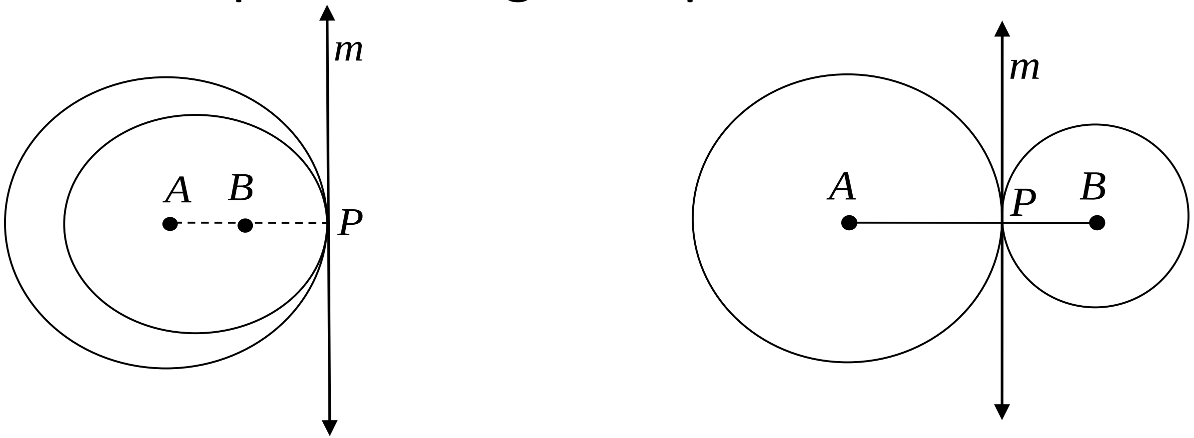
$$(PR)^2 = (PE)^2 + (ER)^2$$

$$17^2 = 8^2 + 15^2$$

289 = 289

Since $(PR)^2 = (PE)^2 + (ER)^2$, △ PRE is a right triangle. So, ∠ PER is a right angle, and ER is a tangent to ⊙ P at E.

Theorem 4: If two circles are tangent internally or externally, their lines of centers pass through the point contact.



Given: ⊙ A and ⊙ B are tangents to *m* at P.
Prove: AB passes through P.

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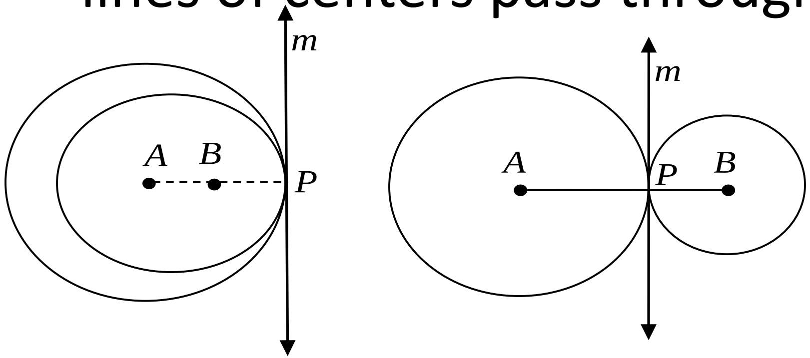

Given: ⊙ A and ⊙ B are tangents to *m* at P.
Prove: AB passes through P.

Statement	Reasons
⊙ A and ⊙ B are tangents to *m* at P.	Given
Draw BP and AP	The line postulate
AP ⊥ m, at BP ⊥ m	The Tangent Line Theorem
AP and BP lie on one line.	Through any point, only one perpendicular to any line can be drawn.
AB PASSES Through P.	One and only one line can be drawn through two points