

# RATIOS AND PROPORTIONS Guide Notes

**RATIO** is a comparison of a number  $a$  and  $b$  using division, where  $b \neq 0$ . It is usually expressed in simplest form and can be expressed as:

$a$  to  $b$

$a:b$

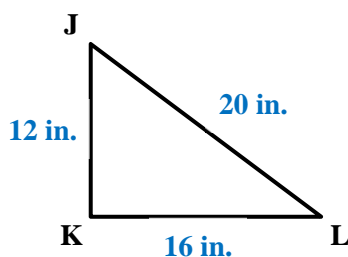
$\frac{a}{b}$

**EXTENDED RATIOS** are ratios that can be used to compare three or more quantities.

$a:b:c$

$a:b:c:d$

**Example:** Express ratio of sides of the triangle in simplest form.



$\overline{JK} : \overline{KL} : \overline{JL}$

→  $12 \text{ in.} : 16 \text{ in.} : 20 \text{ in.}$

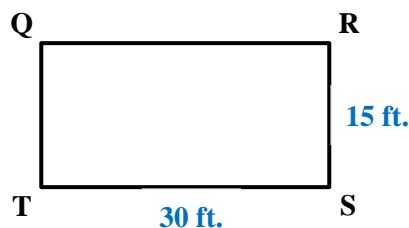
→  $\frac{12 \text{ in.}}{4} : \frac{16 \text{ in.}}{4} : \frac{20 \text{ in.}}{4}$

→  $\frac{12 \text{ in.}}{4 \text{ in.}} : \frac{16 \text{ in.}}{4 \text{ in.}} : \frac{20 \text{ in.}}{4 \text{ in.}}$

→  **$3 : 4 : 5$**

**EQUIVALENT RATIOS** are ratios that have the same simplified form.

**Example:** Express ratio of width and height of the rectangle in simplest form.



$\frac{\text{width of the rectangle}}{\text{height of the rectangle}} = \frac{30 \text{ ft.}}{15 \text{ ft.}} = \frac{2}{1}$

**Sample Problem 1:** Simplify.

a. 35 to 7

$\frac{35}{7}$  to  $\frac{7}{7}$  →  **$5$  to  $1$**

b. 45:63

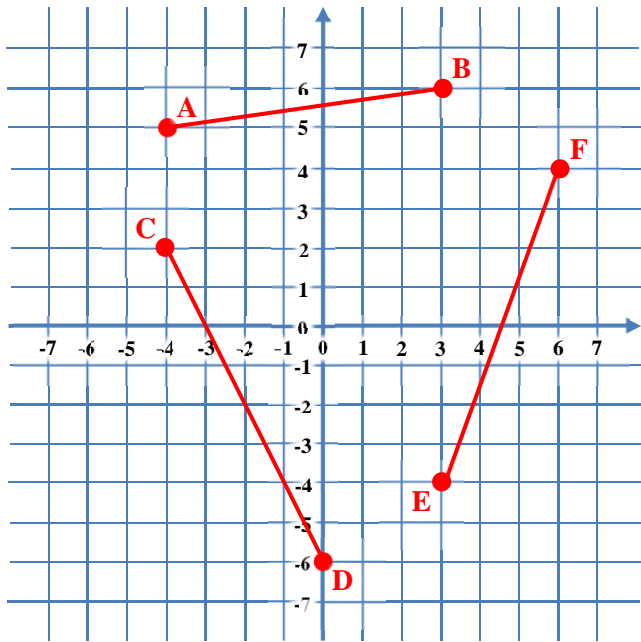
$\frac{45}{9} : \frac{63}{9}$  →  **$5:7$**

c.  $\frac{39}{13}$

$\frac{39}{13} = \frac{13(3)}{13(1)} = \frac{3}{1}$

# RATIOS AND PROPORTIONS Guide Notes

**Sample Problem 2:** Write the ratio expressing the slope of each line segment.



$$a. \quad m_{\overline{AB}} = \frac{y_B - y_A}{x_B - x_A} = \frac{6 - 5}{3 - (-4)} = m_{\overline{AB}} = \frac{1}{7}$$

$A(-4, 5) \qquad B(3, 6)$

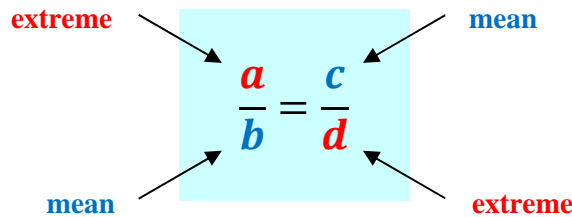
$$b. \quad m_{\overline{CD}} = \frac{y_D - y_C}{x_D - x_C} = \frac{-6 - 2}{0 - (-4)} = \frac{-8}{4} = m_{\overline{CD}} = -2$$

$C(-4, 2) \qquad D(0, -6)$

$$c. \quad m_{\overline{EF}} = \frac{y_F - y_E}{x_F - x_E} = \frac{4 - (-4)}{6 - 3} = m_{\overline{EF}} = \frac{8}{3}$$

$E(3, -4) \qquad F(6, 4)$

**PROPORTION** is an equation stating that two ratios are equal.



**Extremes** are the first and last positions in the proportion.

**Means** are the two middle positions in the proportion.

**Sample Problem 3:** Solve each proportion.

a.  $\frac{x}{11} = \frac{8}{-16}$

$$\frac{x}{11} = \frac{1}{-2} \rightarrow -2x = 1(11)$$

$$\frac{-2x}{-2} = \frac{88}{-2} \rightarrow x = -44$$

b.  $\frac{-6}{9} = \frac{8}{2y + 7}$

$$-6(2y + 7) = 8(9) \rightarrow -12y - 42 = 72$$

$$-12y - 42 + 42 = 72 + 42$$

$$-12y = 114 \rightarrow \frac{-12y}{-12} = \frac{114}{-12}$$

$$y = -\frac{19}{2}$$

**RATIOS AND PROPORTIONS** Guide Notes

c.  $\frac{n+4}{3} = \frac{5n}{6}$

$$6(n+4) = 3(5n) \rightarrow 6n + 24 = 15n$$

$$6n - 6n + 24 = 15n - 6n$$

$$24 = 9n \rightarrow \frac{24}{9} = \frac{9n}{9} \rightarrow \frac{8}{3} = n$$

d.  $\frac{z+2}{11} = \frac{z-2}{15}$

$$\frac{z+1}{11} = \frac{z-2}{15} \rightarrow 15(z+2) = 11(z-2)$$

$$15z + 30 = 11z - 22$$

$$15z - 11z + 30 = 11z - 11z - 22$$

$$4z + 30 = -22$$

$$4z + 30 - 30 = -22 - 30$$

$$4z = -52 \rightarrow \frac{4z}{4} = \frac{-52}{4}$$

$$z = -13$$

**PROPERTIES OF PROPORTIONS****A. Cross Products Property**

In a proportion, the product of the extremes is equal to the product of the means.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } d = bc. \quad (b \neq 0 \text{ and } d \neq 0)$$

**B. Reciprocal Property**

If two ratios are equal, then their reciprocals are also equal.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}. \quad (a \neq 0 \text{ and } c \neq 0)$$

C. If you interchange the means of a proportion, then you form another true proportion.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}. \quad (c \neq 0 \text{ and } d \neq 0)$$

D. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}. \quad (b \neq 0 \text{ and } d \neq 0)$$

**RATIOS AND PROPORTIONS** Guide Notes**Sample Problem 4:** Solve each proportion.

a.  $\frac{3}{7} = \frac{x}{49}$

$$\frac{3(49)}{7} = x \rightarrow \frac{3(7)}{1} = x \rightarrow \mathbf{21 = x}$$

c.  $\frac{s}{8} = \frac{16}{2}$

$$\frac{s}{8} = \frac{16}{2} \rightarrow \frac{s}{8} = \frac{8}{1} \rightarrow s = \frac{8(8)}{1}$$
$$\mathbf{s = 64}$$

b.  $\frac{2n}{25} = \frac{10}{5n}$

$$\frac{2n}{25} = \frac{10}{5n} \rightarrow \frac{2n}{25} = \frac{2}{n}$$

$$2n(n) = 2(25) \rightarrow 2n^2 = 50$$

$$\frac{2n^2}{2} = \frac{50}{2} \rightarrow n^2 = 25 \rightarrow n^2 = 25$$

$$\mathbf{n = \pm 5}$$

d.  $\frac{y+2}{4} = \frac{16}{y+2}$

$$\frac{y+2}{4} = \frac{16}{y+2} \rightarrow (y+2)^2 = 16(4)$$

$$(y+2)^2 = 64 \rightarrow y+2 = \pm 8$$

$$y+2 = 8 \qquad y+2 = -8$$

$$y+2 - 2 = 8 - 2 \qquad y+2 - 2 = -8 - 2$$

$$\mathbf{y = 6}$$

$$\mathbf{y = -10}$$