Conditions of Rhombuses, Rectangles and Squares

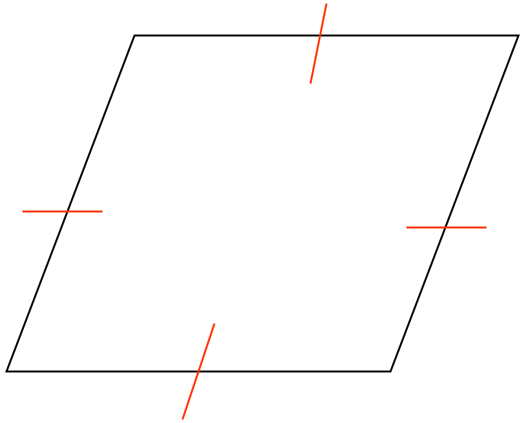
UNIT 6 LESSON 5

Objectives:

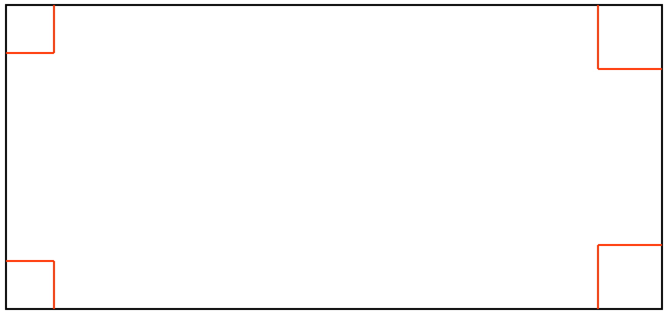
- Use properties of sides and angles of rhombuses, rectangles, and squares.
- Use properties of diagonals of rhombuses, rectangles and squares.

Properties of Special Parallelograms

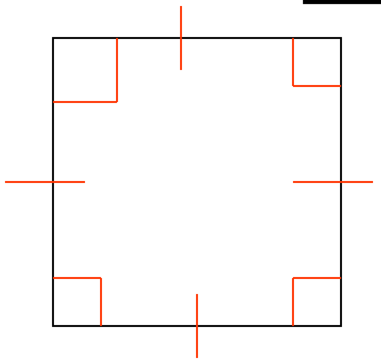
- In this lesson, you will study three special types of parallelograms: rhombuses, rectangles and squares.



A rhombus is a parallelogram with four congruent sides



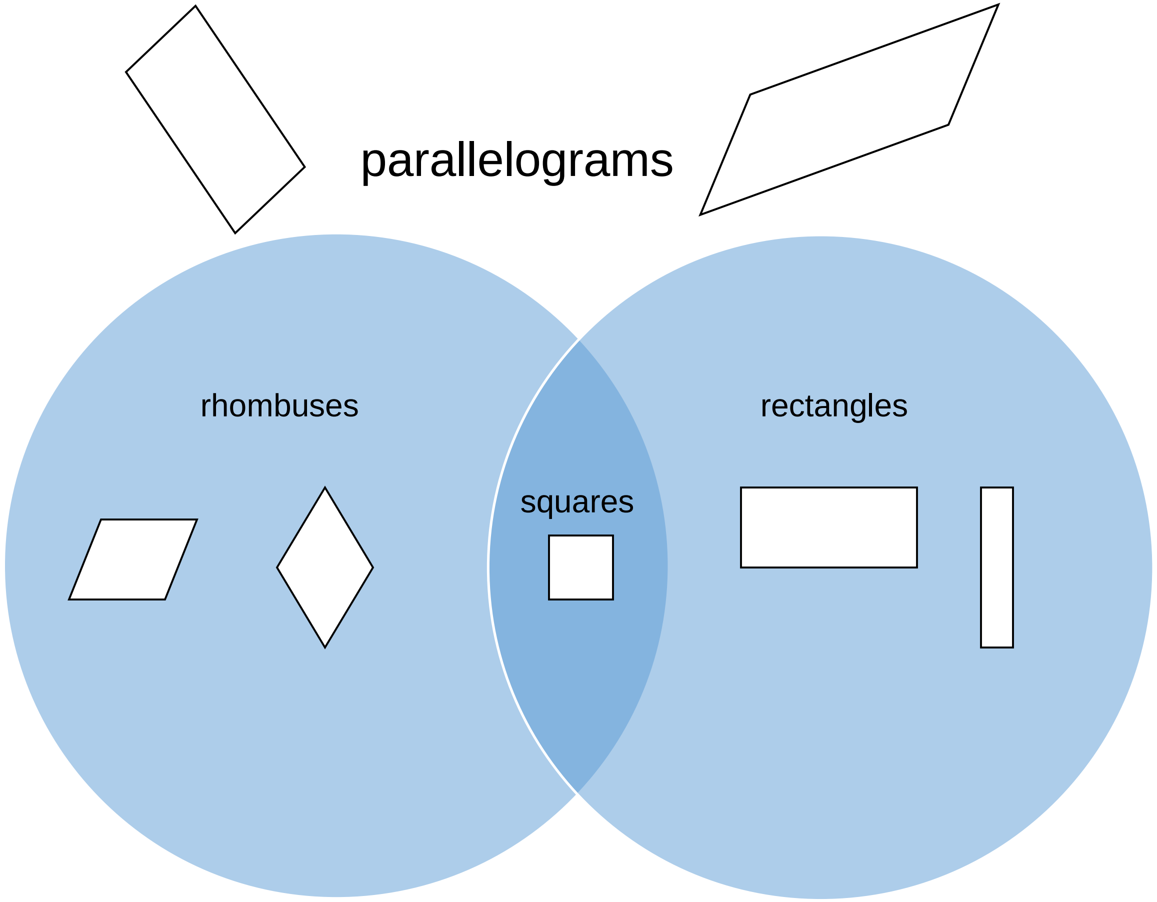
A rectangle is a parallelogram with four right angles.



A square is a parallelogram with four congruent sides and four right angles.

Venn Diagram shows relationships-- MEMORIZE

- Each shape has the properties of every group that it belongs to. For instance, a square is a rectangle, a rhombus and a parallelogram; so it has all of the properties of those shapes.

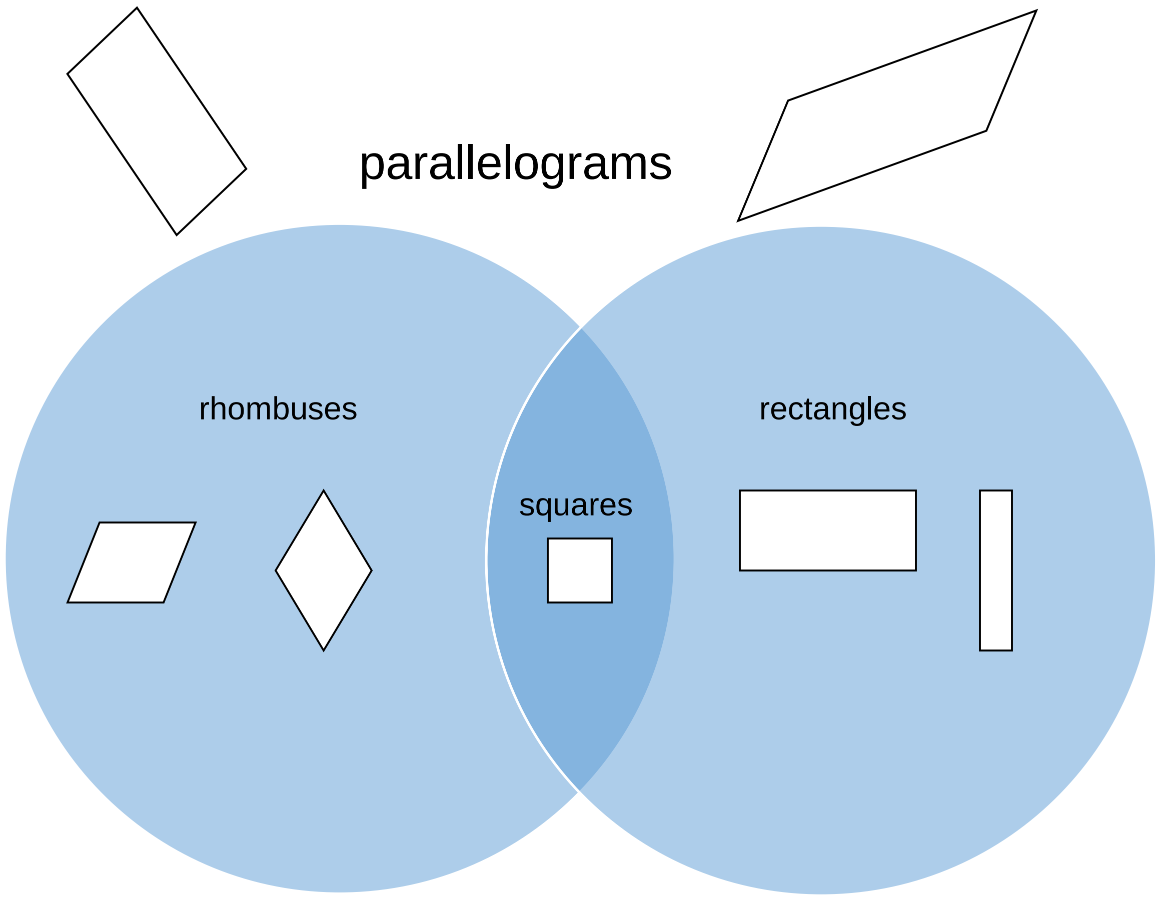


Ex. 1: Describing a special parallelogram

- Decide whether the statement is always, sometimes, or never true.

a. A rhombus is a rectangle.

b. A parallelogram is a rectangle.

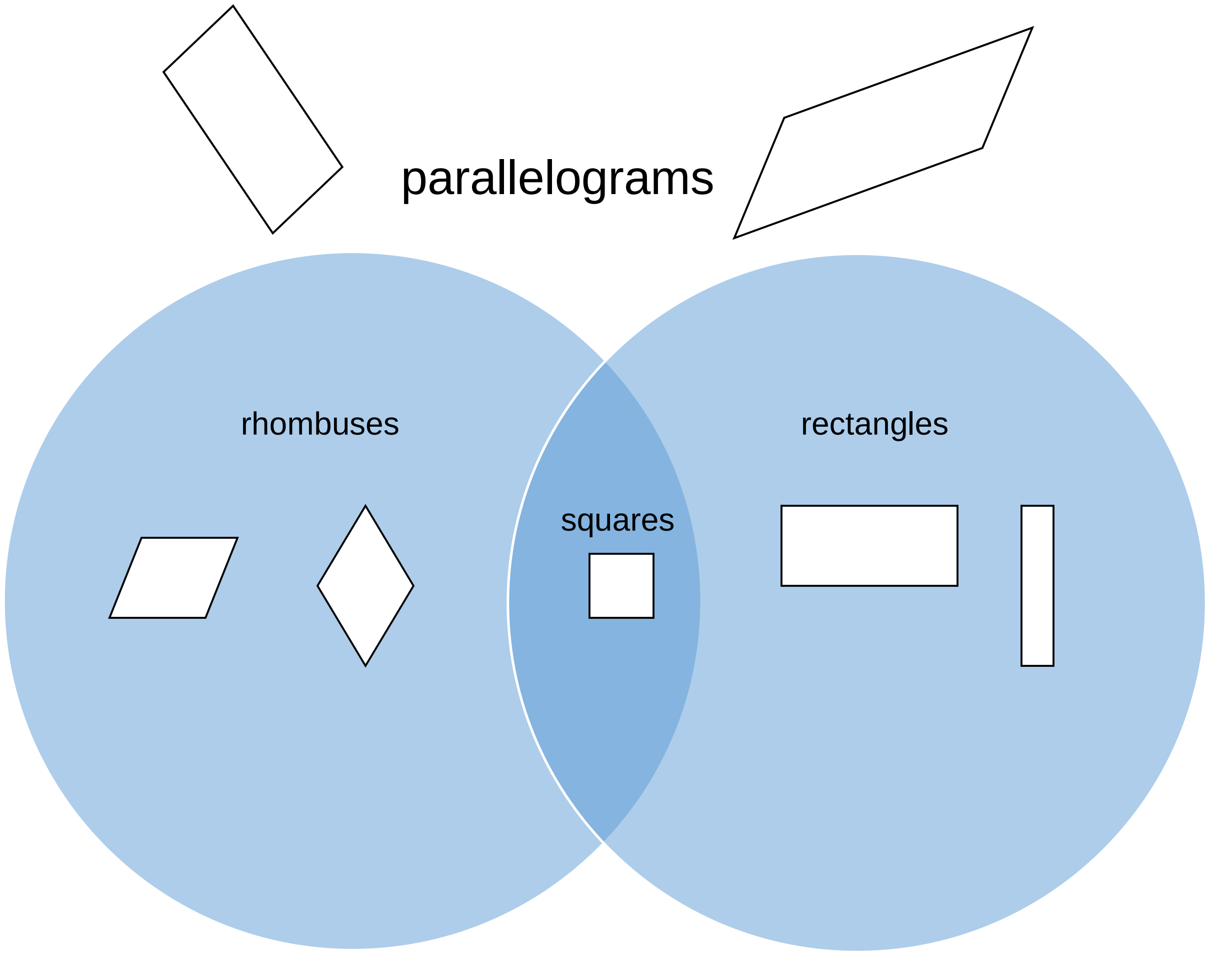


Ex. 1: Describing a special parallelogram

- Decide whether the statement is always, sometimes, or never true.

a. A rhombus is a rectangle.

The statement is sometimes true. In the Venn diagram, the regions for rhombuses and rectangles overlap. IF the rhombus is a square, it is a rectangle.

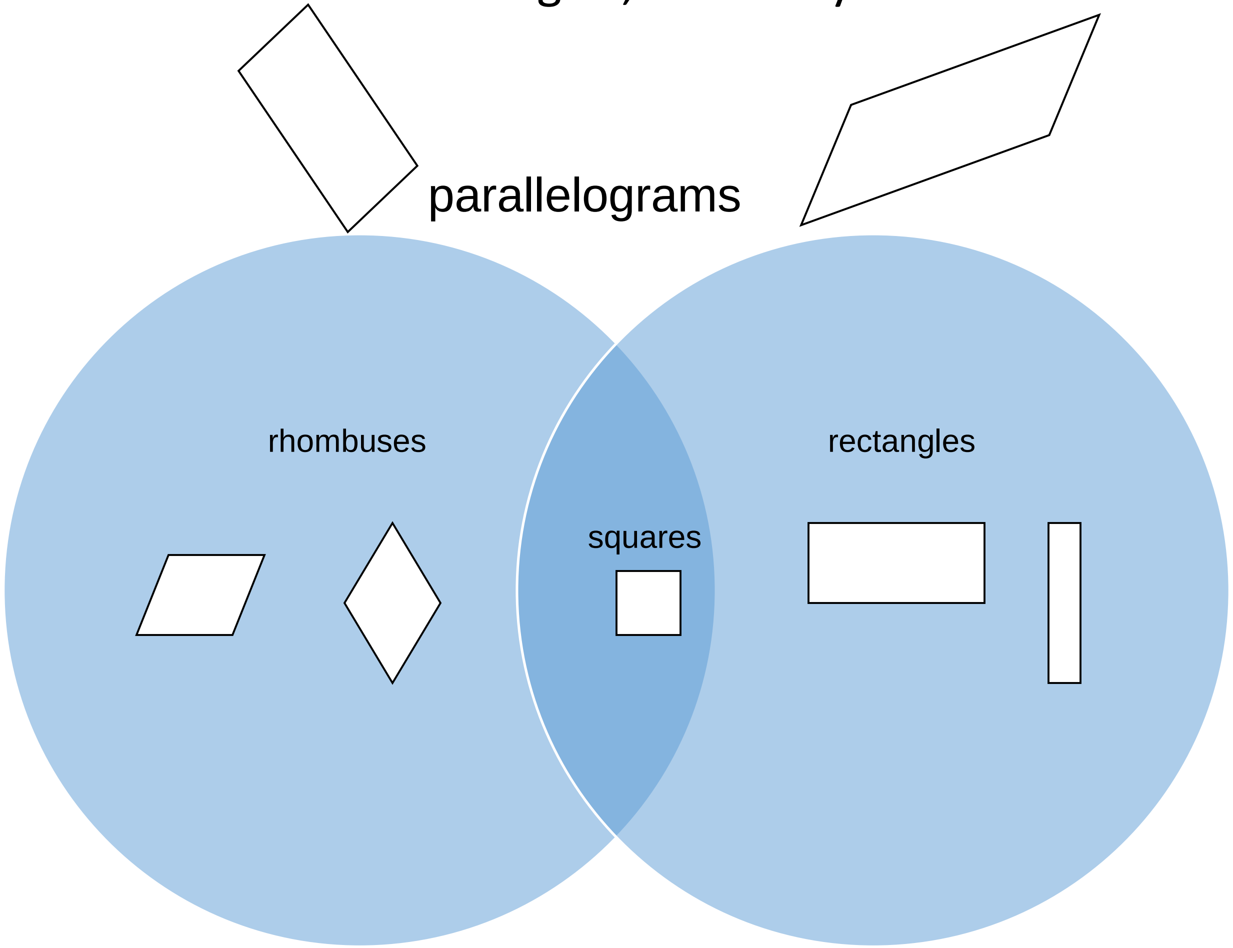


Ex. 1: Describing a special parallelogram

- Decide whether the statement is always, sometimes, or never true.

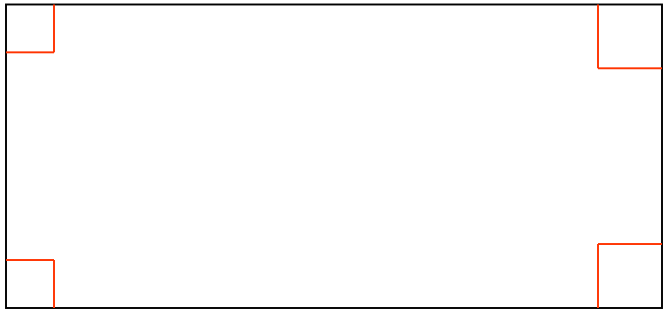
b. A parallelogram is a rectangle.

The statement is sometimes true. Some parallelograms are rectangles. In the Venn diagram, you can see that some of the shapes in the parallelogram box are in the area for rectangles, but many aren't.



Ex. 2: Using properties of special parallelograms

- ABCD is a rectangle. What else do you know about ABCD?



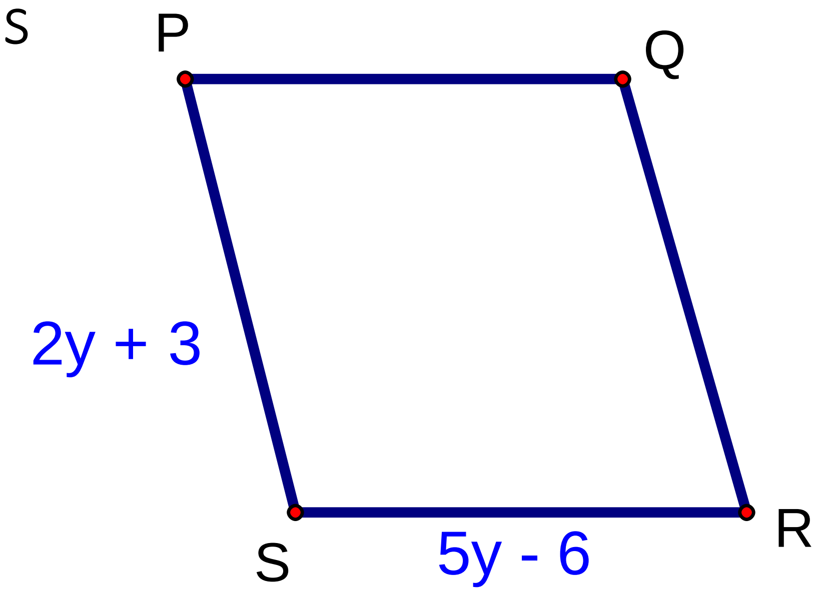
- Because ABCD is a rectangle, it has four right angles by definition. The definition also states that rectangles are parallelograms, so ABCD has all the properties of a parallelogram:
  - Opposite sides are parallel and congruent.
  - Opposite angles are congruent and consecutive angles are supplementary.
  - Diagonals bisect each other.

Take note:

- A rectangle is defined as a parallelogram with four right angles. But any quadrilateral with four right angles is a rectangle because any quadrilateral with four right angles is a parallelogram.
- Corollaries about special quadrilaterals:
  - Rhombus Corollary: A quadrilateral is a rhombus if and only if it has four congruent sides.
  - Rectangle Corollary: A quadrilateral is a rectangle if and only if it has four right angles.
  - Square Corollary: A quadrilateral is a square if and only if it is a rhombus and a rectangle.
  - You can use these to prove that a quadrilateral is a rhombus, rectangle or square without proving first that the quadrilateral is a parallelogram.

Ex. 3: Using properties of a Rhombus

- In the diagram at the right, PQRS is a rhombus. What is the value of y?

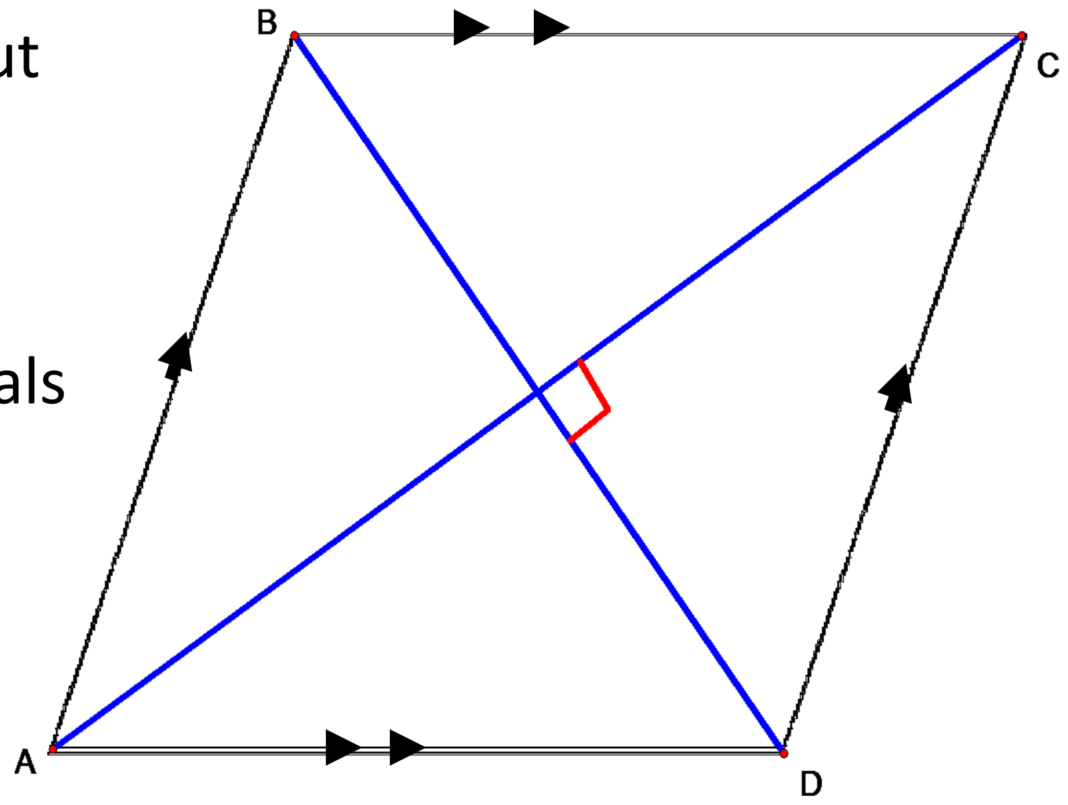


All four sides of a rhombus are ≅, so $\overline{RS} = \overline{PS}$.

$5y - 6 = 2y + 3$	Equate lengths of ≅ sides.
$5y = 2y + 9$	Add 6 to each side.
$3y = 9$	Subtract 2y from each side.
$y = 3$	Divide each side by 3.

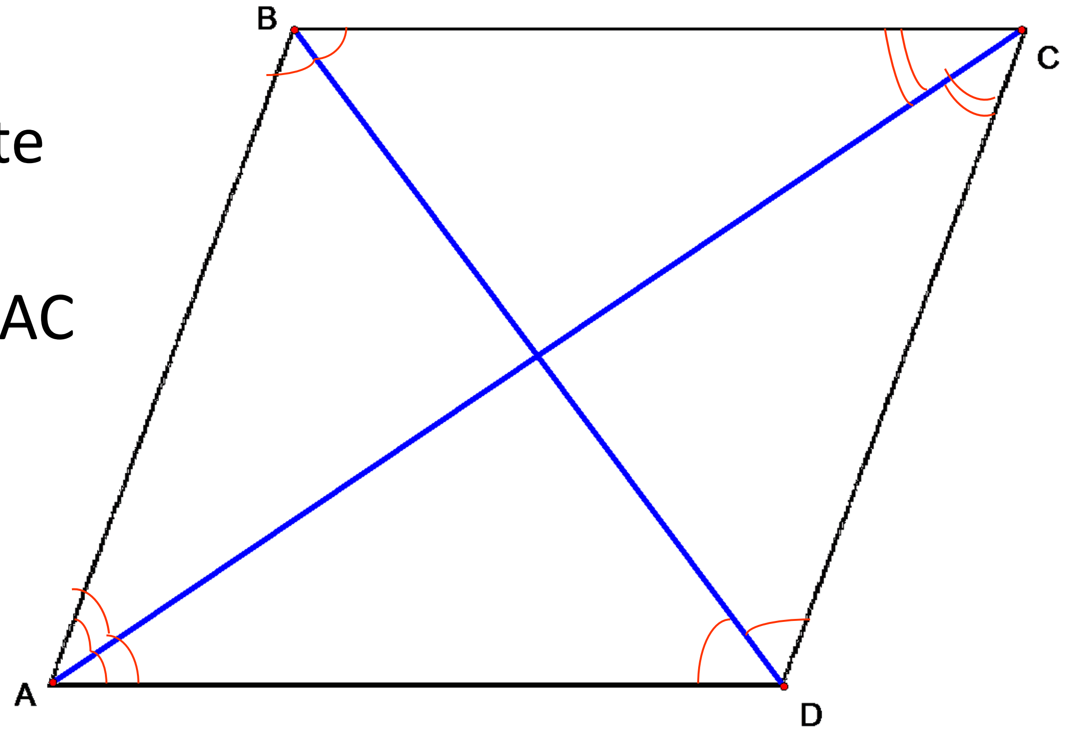
Using diagonals of special parallelograms

- The following theorems are about diagonals of rhombuses and rectangles.
- Theorem 1: A parallelogram is a rhombus if and only if its diagonals are perpendicular.
- ABCD is a rhombus if and only if $AC \perp BD$.



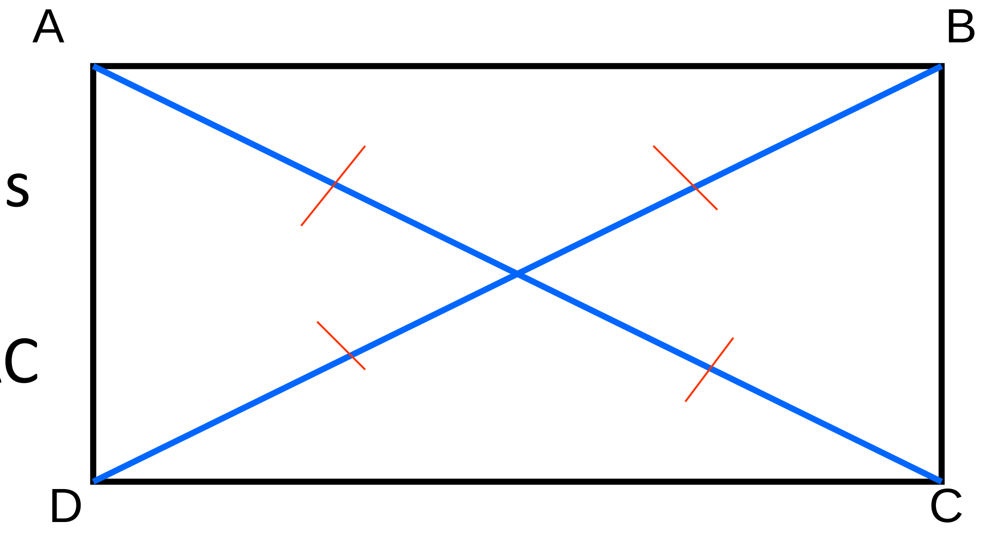
Using diagonals of special parallelograms

- Theorem 2: A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.
- ABCD is a rhombus if and only if AC bisects $\angle DAB$ and $\angle BCD$ and BD bisects $\angle ADC$ and $\angle CBA$.



Using diagonals of special parallelograms

- Theorem 3: A parallelogram is a rectangle if and only if its diagonals are congruent.
- ABCD is a rectangle if and only if AC ≅ BD.



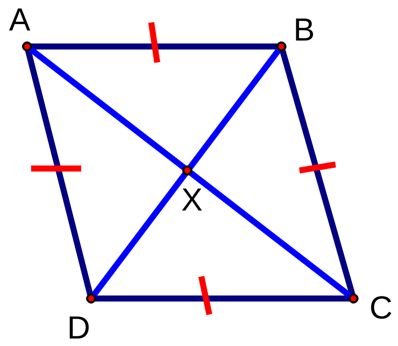
NOTE:

- You can rewrite the fist Theorem as a conditional statement and its converse.
- Conditional statement: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
- Converse: If a parallelogram is a rhombus, then its diagonals are perpendicular.

Ex. 4: Proving Theorem 1

Given: ABCD is a rhombus

Prove: $AC \perp BD$



Statements:	Reasons:
1. ABCD is a rhombus	1. Given
2. $AB \cong CB$	
3. $AX \cong CX$	
4. $BX \cong DX$	
5. $\Delta AXB \cong \Delta CXB$	
6. $\angle AXB \cong \angle CXB$	
7. $AC \perp BD$	

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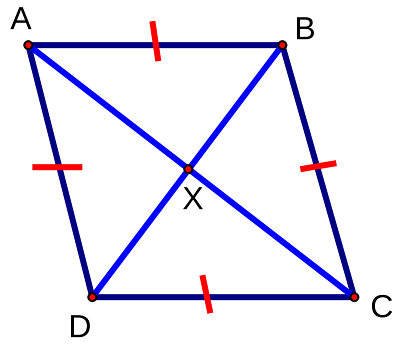

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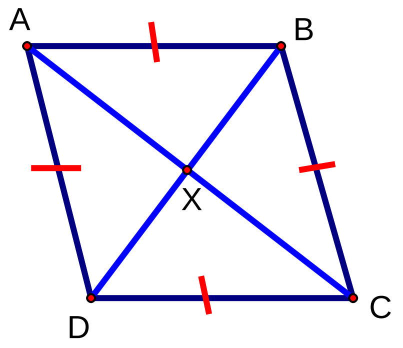

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Statements:	Reasons:
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3. $AX \cong CX$	3. Def. of ▱. Diagonals bisect each other.
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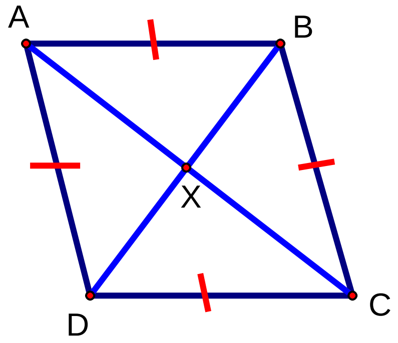

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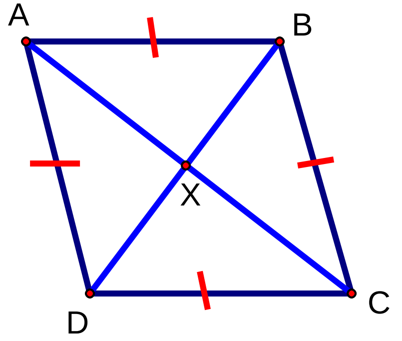

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5. $\Delta AXB \cong \Delta CXB$	5. SSS congruence post.
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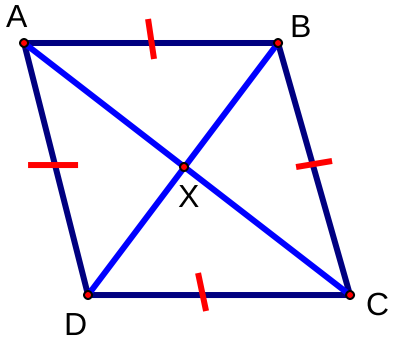
Prove: $AC \perp BD$



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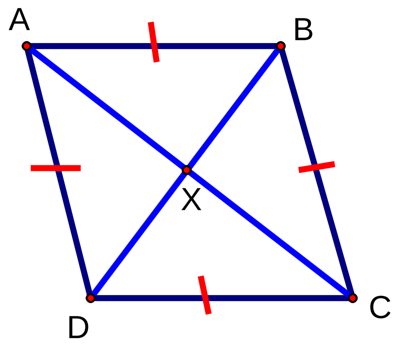
6. $\angle AXB \cong \angle CXB$	6. CPCTC
7. $AC \perp BD$	

Ex. 4: Proving Theorem 1

Given: ABCD is a rhombus

Prove: $AC \perp BD$



Statements:	Reasons:
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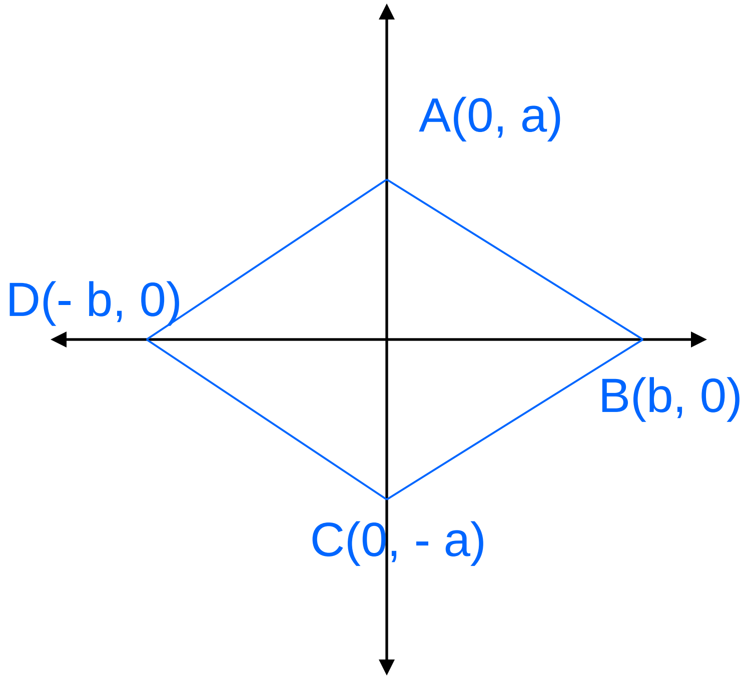
5. $\Delta AXB \cong \Delta CXB$	5. SSS congruence post.
6. $\angle AXB \cong \angle CXB$	6. CPCTC
7. $AC \perp BD$	7. Congruent Adjacent $\angle$s

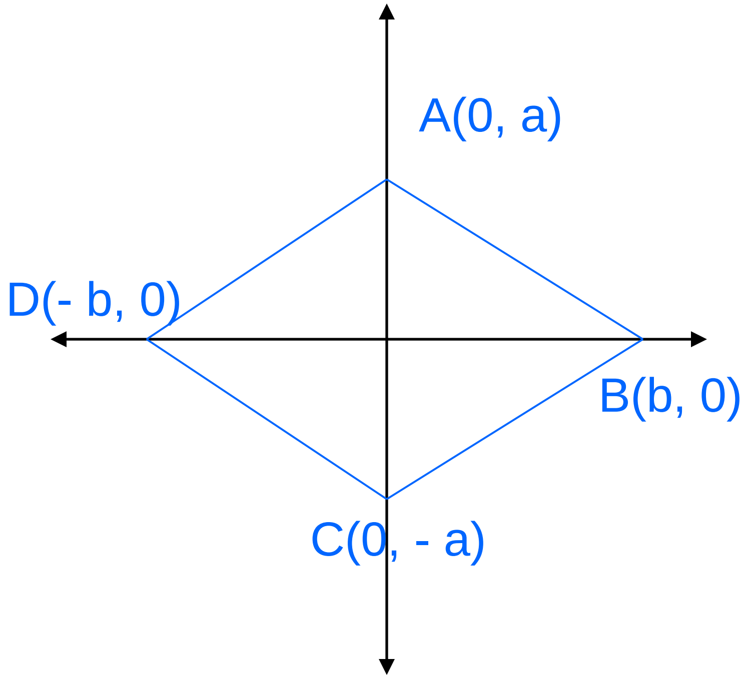
Ex. 5: Coordinate Proof of Theorem 1
Given: ABCD is a parallelogram, AC ⊥ BD.
Prove: ABCD is a rhombus

- Assign coordinates. Because AC⊥ BD, place ABCD in the coordinate plane so AC and BD lie on the axes and their intersection is at the origin.
- Let (0, a) be the coordinates of A, and let (b, 0) be the coordinates of B.
- Because ABCD is a parallelogram, the diagonals bisect each other and OA = OC. So, the coordinates of C are (0, - a). Similarly the coordinates of D are (- b, 0).



Ex. 5: Coordinate Proof of Theorem 1
Given: ABCD is a parallelogram, $AC \perp BD$.
Prove: ABCD is a rhombus

- Find the lengths of the sides of ABCD. Use the distance formula (See – you're never going to get rid of this)
- $AB = \sqrt{(b-0)^2 + (0-a)^2} = \sqrt{b^2 + a^2}$
- $BC = \sqrt{(0-b)^2 + (-a-0)^2} = \sqrt{b^2 + a^2}$
- $CD = \sqrt{(-b-0)^2 + [0-(-a)]^2} = \sqrt{b^2 + a^2}$
- $DA = \sqrt{[(0-(-b)]^2 + (a-0)^2} = \sqrt{b^2 + a^2}$



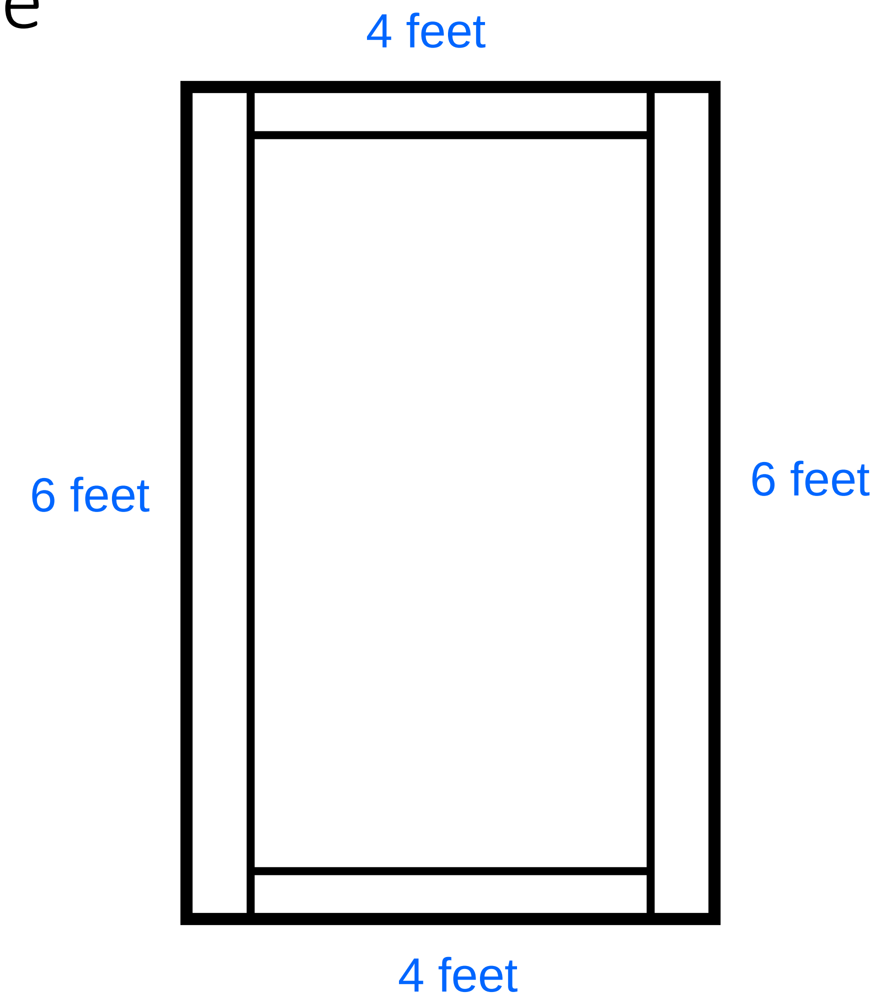
All the side lengths are equal, so ABCD is a rhombus.

Ex 6: Checking a rectangle

- CARPENTRY. You are building a rectangular frame for a theater set.

a. First, you nail four pieces of wood together as shown at the right. What is the shape of the frame?

b. To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches. The other is 7 feet 2 inches. Is the frame a rectangle? Explain.



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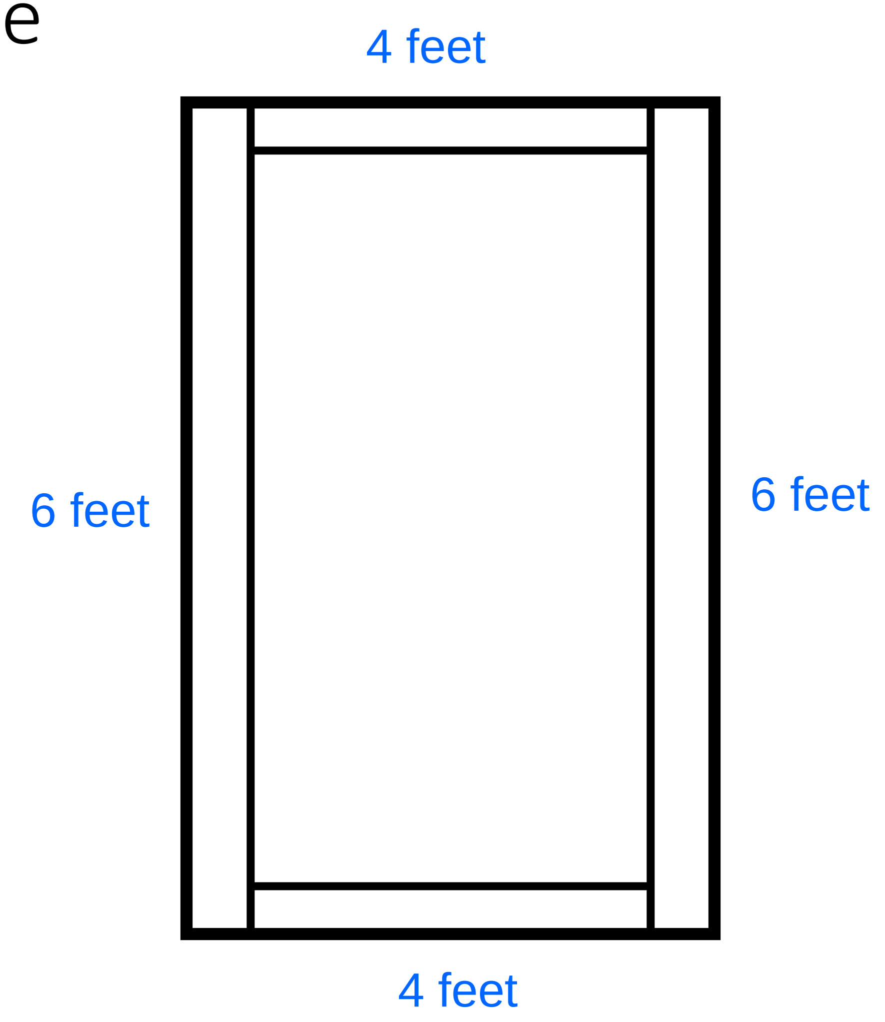

Opposite sides are congruent, so the frame is a parallelogram.

Ex 6: Checking a rectangle

b. To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches. The other is 7 feet 2 inches. Is the frame a rectangle? Explain.

The parallelogram is NOT a rectangle. If it were a rectangle, the diagonals would be congruent.