INEQUALITIES IN ONE TRIANGLE

Students will be able to:
Apply inequalities in one triangle.

Key Vocabulary:

• Angle – Side Theorem
• Converse of Angle – side Theorem
• Exterior angle Inequality Theorem
• Triangle Inequality Theorem
INEQUALITIES IN ONE TRIANGLE

ANGLE – SIDE THEOREM

“If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.”

- If $AB > BC$ then $\angle ACB > \angle BAC$.
- If $AC > BC$ then $\angle ABC > \angle BAC$.
- If $AC > AB$ then $\angle ABC > \angle ACB$. 

A

\[ \text{B} \]

\[ \text{C} \]
Sample Problem 1: Write the angles in order from smallest to largest.
Sample Problem 1: Write the angles in order from smallest to largest.

\[ AB = 12.13 \] is opposite \( \angle C \)

\[ BC = 10.3 \] is opposite \( \angle A \)

\[ AC = 6.4 \] is opposite \( \angle B \)

\[ AC < BC < AB \]

\[ m\angle B < m\angle A < m\angle C \]
CONVERSE OF ANGLE – SIDE THEOREM

“If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.”

- If $\angle ACB > \angle BAC$ then $\overline{AB} > \overline{BC}$.
- If $\angle ABC > \angle BAC$ then $\overline{AC} > \overline{BC}$.
- If $\angle ABC > \angle ACB$ then $\overline{AC} > \overline{AB}$. 
Sample Problem 2: Write the sides in order from shortest to longest.
Sample Problem 2: Write the sides in order from shortest to longest.

\( AB \) is opposite \( m\angle C = 105^\circ \)

\( BC \) is opposite \( m\angle A = 42^\circ \)

\( AC \) is opposite \( m\angle B = 33^\circ \)

\( m\angle B < m\angle A < m\angle C \)

\( AC < BC < AB \)
INEQUALITIES IN ONE TRIANGLE

EXTERIOR ANGLE INEQUALITY THEOREM

“The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.”

\[ \angle ABD > \angle BAC \]

and

\[ \angle ABD > \angle BCA \]
Sample Problem 3: Determine the smallest and the largest angles.
Sample Problem 3: Determine the smallest and the largest angles.

**Interior angles:** $m\angle 5 > m\angle 3 > m\angle 2$

**Exterior angles:** $m\angle 1 > m\angle 4 > m\angle 6$

Smallest angle: $m\angle 2$

Largest angle: $m\angle 1$
INEQUALITIES IN ONE TRIANGLE

TRIANGLE INEQUALITY THEOREM

“The sum of the lengths of any two sides of a triangle is greater than the length of the third side.”

\[ \overline{AB} + \overline{BC} > \overline{AC} \]

\[ \overline{AC} + \overline{BC} > \overline{AB} \]

\[ \overline{AB} + \overline{AC} > \overline{BC} \]
Sample Problem 4: A triangle has one side of length 12 and another of length 8. Identify the possible lengths of the third side.
**Sample Problem 4:** A triangle has one side of length 12 and another of length 8. Identify the possible lengths of the third side.

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>( x = 12 )</td>
<td>( y = 8 )</td>
<td>( 4 &lt; z &lt; 20 )</td>
</tr>
<tr>
<td>( x + y &gt; z )</td>
<td>( x + z &gt; y )</td>
<td>( y + z &gt; x )</td>
</tr>
<tr>
<td>( 12 + 8 &gt; z )</td>
<td>( 12 + z &gt; 8 )</td>
<td>( 8 + z &gt; 12 )</td>
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<tr>
<td>( 20 &gt; z )</td>
<td>( z &gt; 8 - 12 )</td>
<td>( z &gt; 12 - 8 )</td>
</tr>
<tr>
<td></td>
<td>( z &gt; -4 )</td>
<td>( z &gt; 4 )</td>
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