Isosceles, Equilateral, and Right Triangles

UNIT 4 LESSON 5

Objectives:

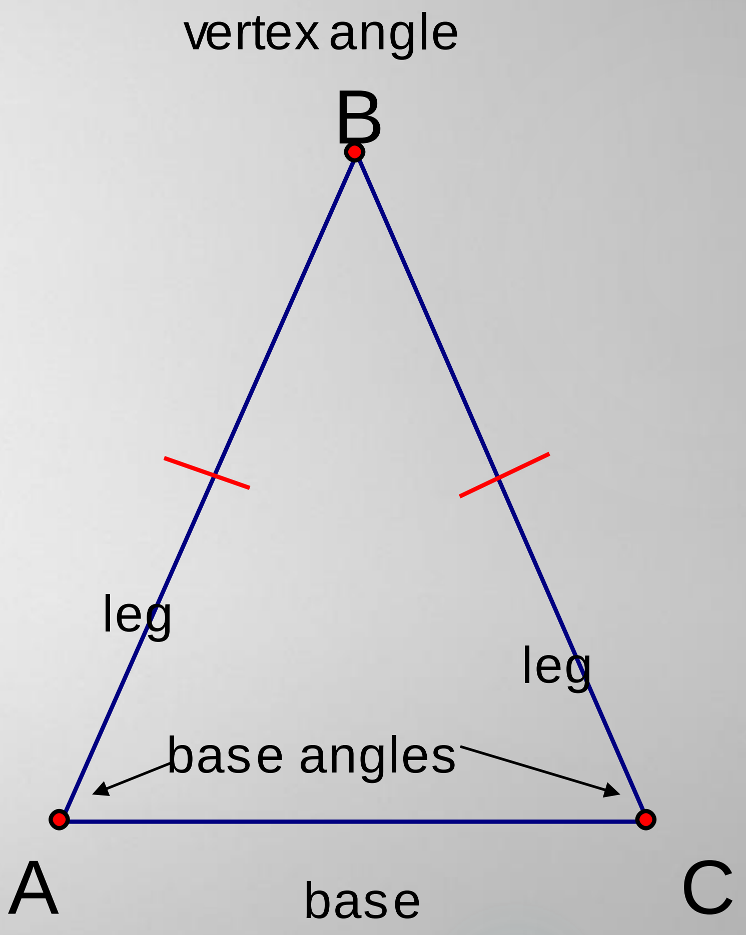
- Use properties of isosceles and equilateral triangles
- Use properties of right triangles

Assignment:

- pp. 239-241 #1-26, 29-32, 33, 39

Using properties of Isosceles Triangles

- A triangle is an isosceles if it has at least two congruent sides. If it has exactly two congruent sides, then they are the legs of the triangle and the non-congruent side is the base. The two angles adjacent to the base are the base angles. The angle opposite the base is the vertex angle.



Investigating Isosceles Triangles

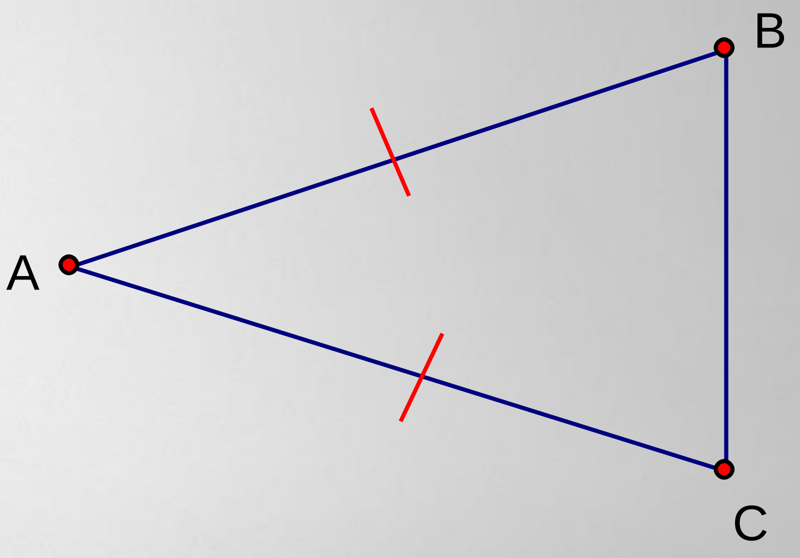
1. Use a straight edge and a compass to construct an acute isosceles triangle. Then fold the triangle along a line that bisects the vertex angle as shown.
2. Repeat the procedure for an obtuse isosceles triangle.
3. What observations can you make about the base angles of an isosceles triangle? Write your observations as a conjecture (what did you observe?).

What did you discover?

- In the activity, you may have discovered the Base Angles Theorem, which is proved in Example 1 which follows this slide. The converse of this theorem is also true.

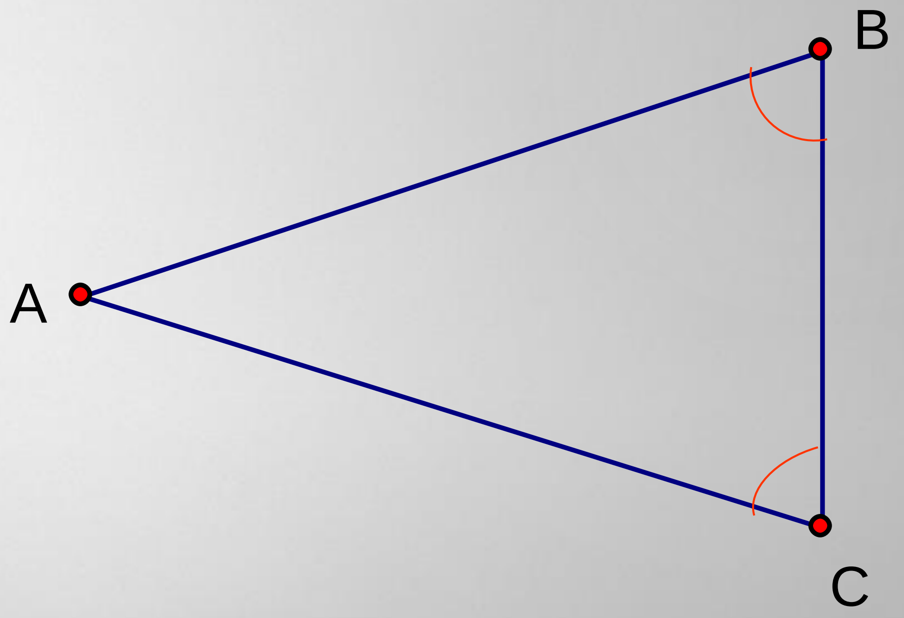
Theorems

- Base Angles Theorem: If two sides of a triangle are congruent, then the angles opposite them are congruent.
- If AB ≅ AC, then ∠B ≅ ∠C.



Theorems

- Converse of the Base Angles Theorem: If two angles of a triangle are congruent, then the sides opposite them are congruent.
- If $\angle B \cong \angle C$, then $AB \cong AC$.

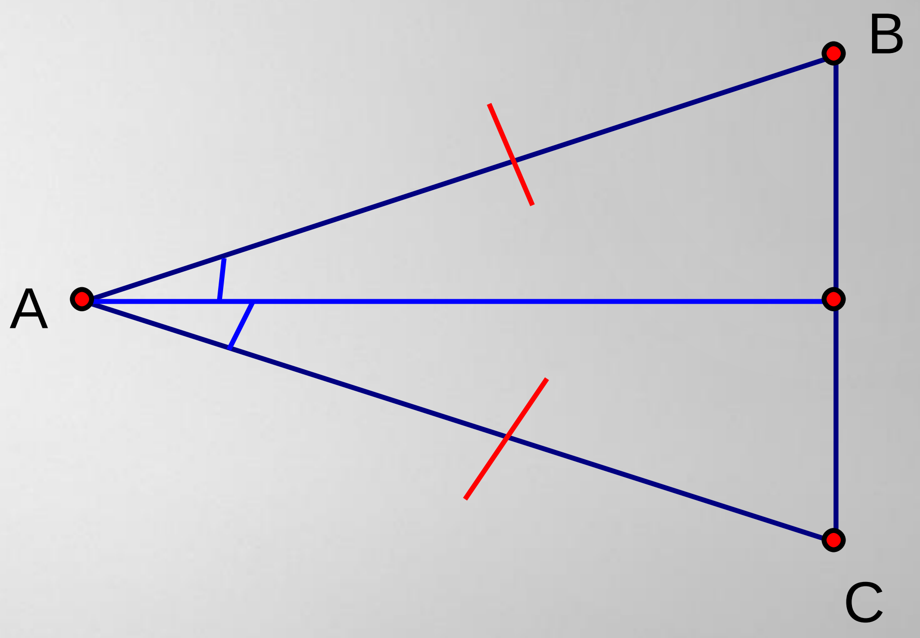


Ex. 1: Proof of the Base Angles Theorem

Given: ΔABC, AB≅AC

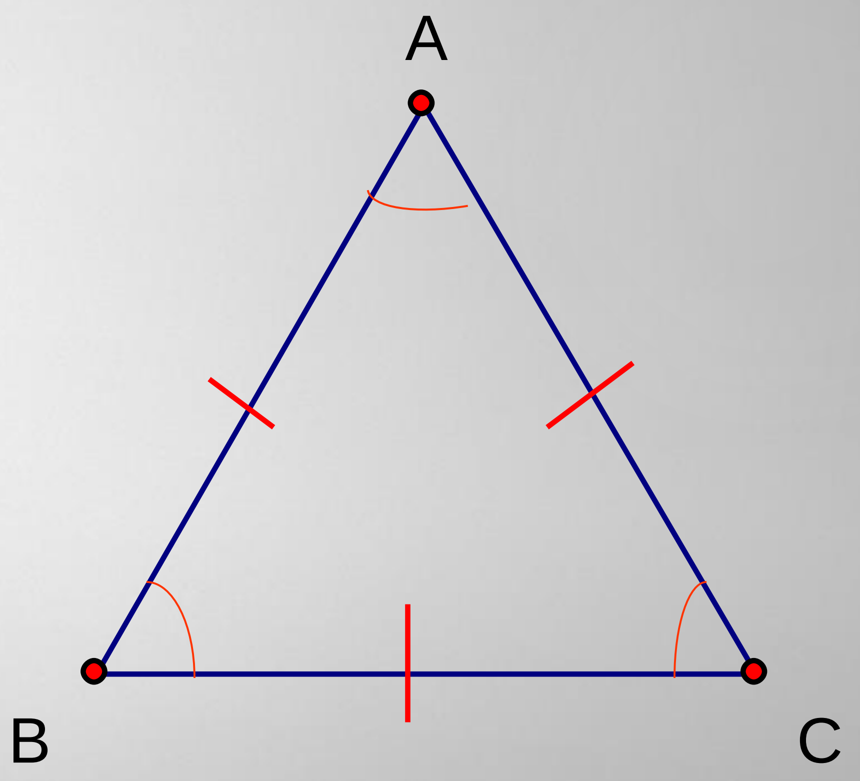
Prove: ∠B ≅ ∠C

Paragraph proof: Draw the bisector of ∠CAB. By construction, ∠CAD ≅ ∠BAD. You are given that AB ≅ AC. Also, DA ≅ DA by the Reflexive property of Congruence. Use the SAS Congruence postulate to conclude that ΔADB≅ΔADC. Because CPCTC, it follows that ∠B ≅ ∠C.



Remember:

- An EQUILATERAL triangle is a special type of isosceles triangle. The corollaries below state that a triangle is EQUILATERAL if and only if it is EQUIANGULAR.
- Corollary to theorem—If a triangle is equilateral, then it is equiangular.
- Corollary to theorem– If a triangle is equiangular, then it is equilateral.



Ex. 2: Using Equilateral and Isosceles Triangles

a. Find the value of x

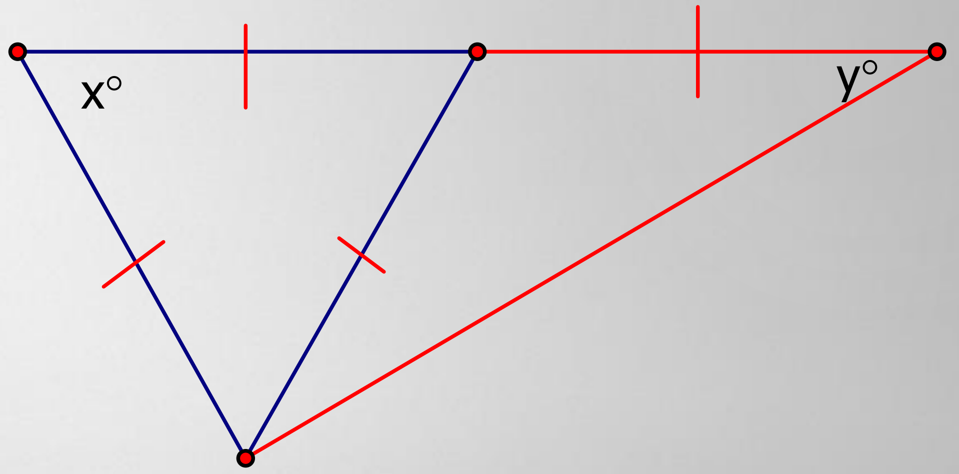
b. Find the value of y

Solution a: How many total degrees in a triangle?

This is an equilateral triangle which means that all three angles are the same.

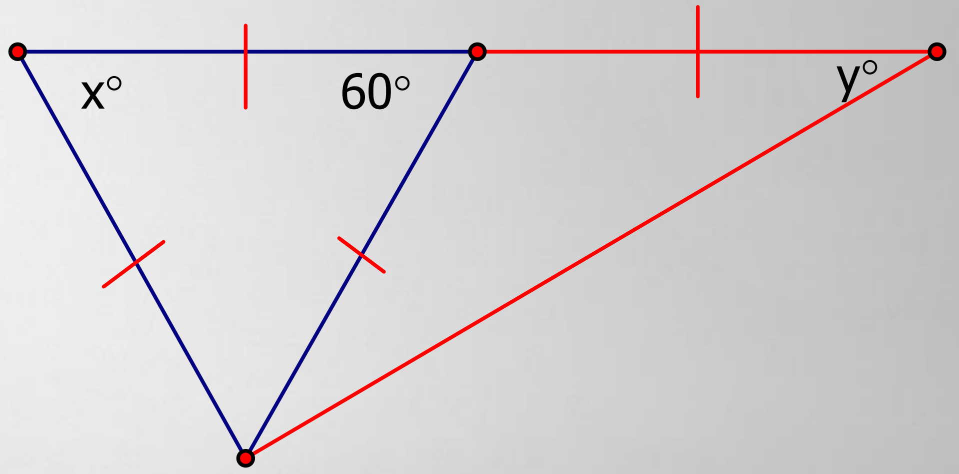
3x = 180 – Triangle Sum Theorem.

X = 60



Ex. 2: Using Equilateral and Isosceles Triangles

a. Find the value of x
b. Find the value of y



Solution b: How many total degrees in a line?

The triangle has base angles of y° which are equal. (Base Angles Theorem). The other base angle has the same measure. The vertex angle forms a linear pair with a 60° angle, so its measure is 120°

$120° + 2y° = 180°$(Triangle Sum Theorem)

$2y = 60$ (Solve for y)

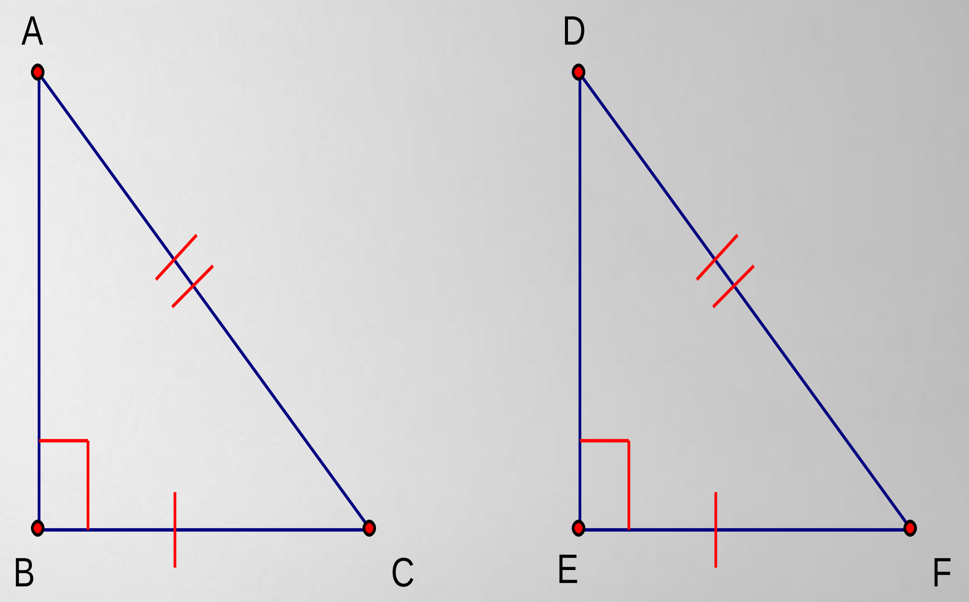
$y = 30$

Using Properties of Right Triangles

- You have learned four ways to prove that triangles are congruent.
  - Side-Side-Side (SSS) Congruence Postulate
  - Side-Angle-Side (SAS) Congruence Postulate
  - Angle-Side-Angle (ASA) Congruence Postulate
  - Angle-Angle-Side (AAS) Congruence Theorem
- The Hypotenuse-Leg Congruence Theorem on the next slide can be used to prove that two RIGHT triangles are congruent.

Hypotenuse-Leg (HL) Congruence Theorem

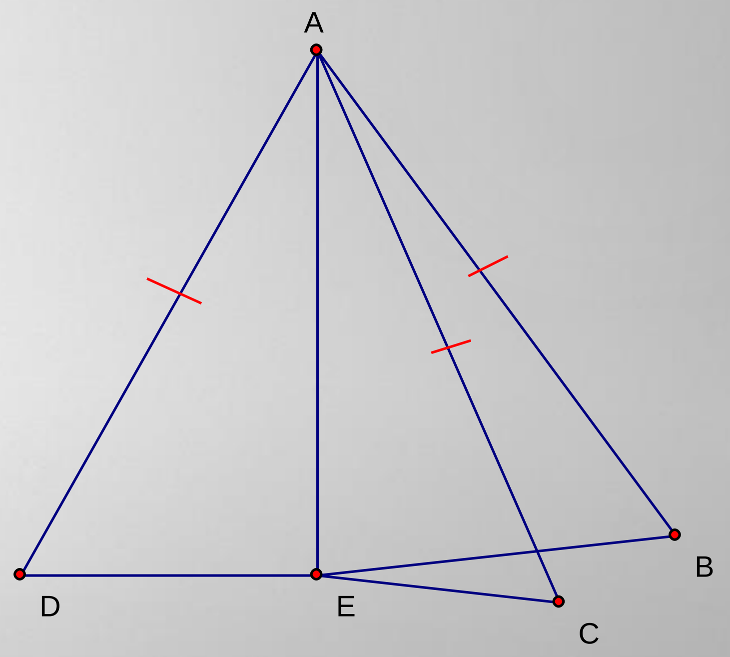
- If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.
- If BC ≅ EF and AC ≅ DF, then ΔABC ≅ ΔDEF.



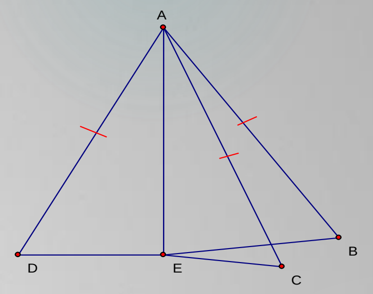
Ex. 3: Proving Right Triangles Congruent

- The television antenna is perpendicular to the plane containing points B, C, D, and E. Each of the stays running from the top of the antenna to B, C, and D uses the same length of cable. Prove that ΔAEB, ΔAEC, and ΔAED are congruent.

Given: $AE \perp EB$, $AE \perp EC$, $AE \perp ED$, $AB \cong AC \cong AD$.
Prove $\Delta AEB \cong \Delta AEC \cong \Delta AED$



Given: AE⊥EB, AE⊥EC, AE⊥ED, AB≅AC≅AD.
Prove ΔAEB ≅ ΔAEC ≅ ΔAED

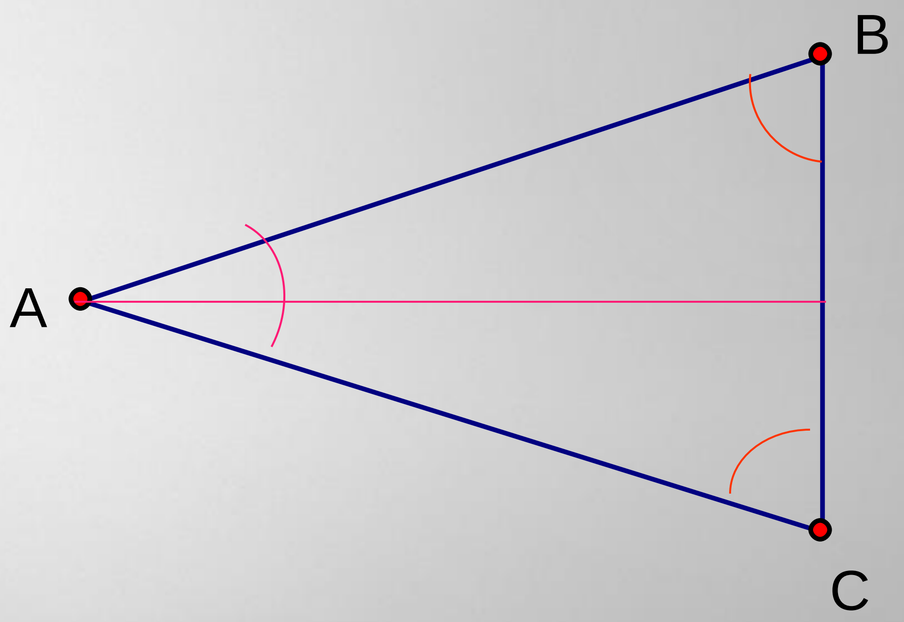


Paragraph Proof: You are given that AE⊥EB, AE⊥EC, which implies that ∠AEB and ∠AEC are right angles. By definition, ΔAEB and ΔAEC are right triangles. You are given that the hypotenuses of these two triangles, AB and AC, are congruent. Also, AE is a leg for both triangles and AE ≅ AE by the Reflexive Property of Congruence. Thus, by the Hypotenuse-Leg Congruence Theorem, ΔAEB ≅ ΔAEC.

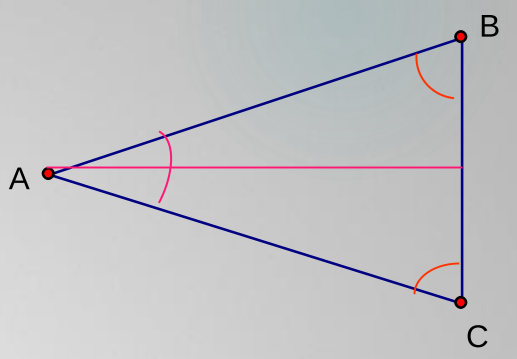
Similar reasoning can be used to prove that ΔAEC ≅ ΔAED. So, by the Transitive Property of Congruent Triangles, ΔAEB ≅ ΔAEC ≅ ΔAED.

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC

- Draw the angle bisector of ∠BAC.

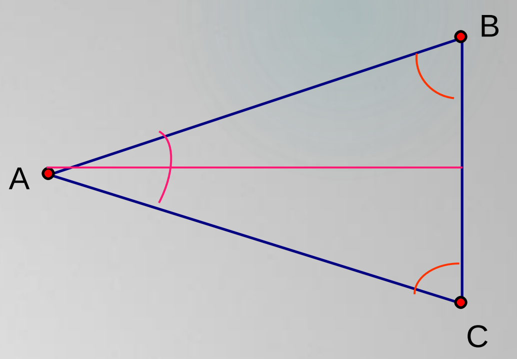


Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



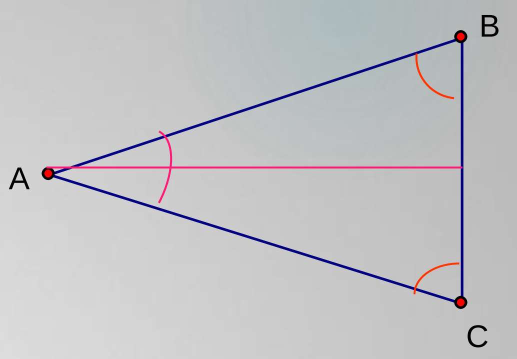
Statements:	Reasons:
1. ∠B ≅ ∠C	1. Given
2. AD is ∠ bisector of ∠A	
3. ∠BAD ≅ ∠CAD	
4. ∠BDA ≅ ∠CDA	
5. AE ≅ AE	
6. ΔBDA ≅ ΔCDA	
7. AB ≅ AC	

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



Statements:	Reasons:
1. ∠B ≅ ∠C	1. Given
2. AD is ∠ bisector of ∠A	2. By construction
3. ∠BAD ≅ ∠CAD	
4. ∠BDA ≅ ∠CDA	
5. AE ≅ AE	
6. ΔBDA ≅ ΔCDA	
7. AB ≅ AC	

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



Statements:	Reasons:
1. ∠B ≅ ∠C	1. Given
2. AD is ∠ bisector of ∠A	2. By construction
3. ∠BAD ≅ ∠CAD	3. Definition ∠ Bisector
4. ∠BDA ≅ ∠CDA	
5. AE ≅ AE	
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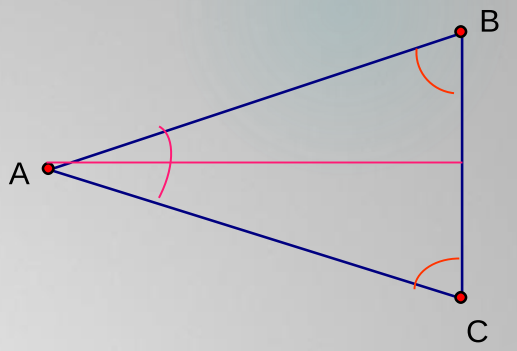
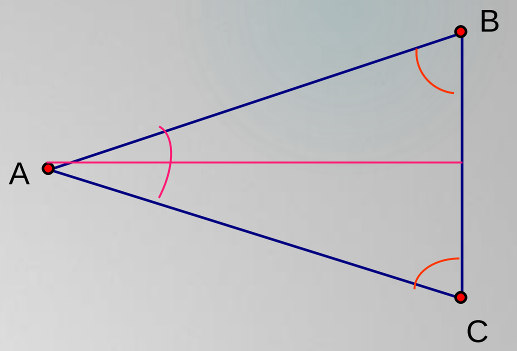
7. AB ≅ AC	

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



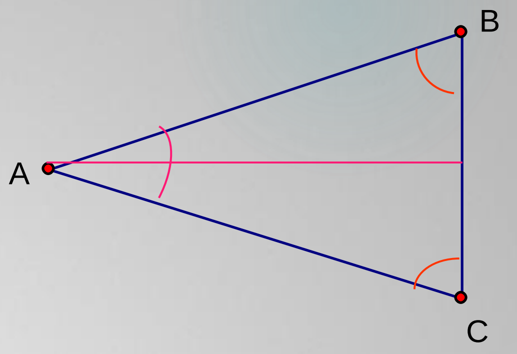
Statements:	Reasons:
1. ∠B ≅ ∠C	1. Given
2. AD is ∠ bisector of ∠A	2. By construction
3. ∠BAD ≅ ∠CAD	3. Definition ∠ Bisector
4. ∠BDA ≅ ∠CDA	4. Third Angles Theorem
5. AE ≅ AE	
6. ΔBDA ≅ ΔCDA	
7. AB ≅ AC	

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



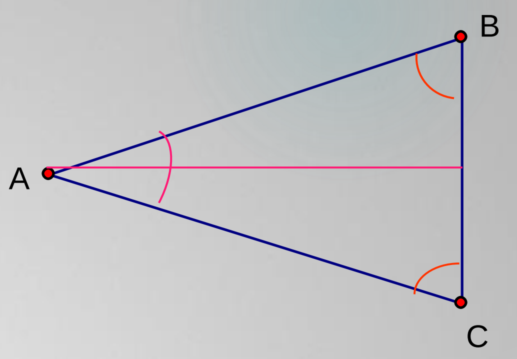
Statements:	Reasons:
1. ∠B ≅ ∠C	1. Given
2. AD is ∠ bisector of ∠A	2. By construction
3. ∠BAD ≅ ∠CAD	3. Definition ∠ Bisector
4. ∠BDA ≅ ∠CDA	4. Third Angles Theorem
5. AE ≅ AE	5. Reflexive Property
6. ΔBDA ≅ ΔCDA	
7. AB ≅ AC	

Proof:
Given: $\angle B \cong \angle C$
Prove: $AB \cong AC$



Statements:	Reasons:
1. $\angle B \cong \angle C$	1. Given
2. AD is $\angle$ bisector of $\angle A$	2. By construction
3. $\angle BAD \cong \angle CAD$	3. Definition $\angle$ Bisector
4. $\angle BDA \cong \angle CDA$	4. Third Angles Theorem
5. $AE \cong AE$	5. Reflexive Property
6. $\Delta BDA \cong \Delta CDA$	6. ASA Congruence Postulate
7. $AB \cong AC$	

Proof:
Given: ∠B ≅ ∠C
Prove: AB ≅ AC



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5. AE ≅ AE	5. Reflexive Property
6. ΔBDA ≅ ΔCDA	6. ASA Congruence Postulate
7. AB ≅ AC	7. CPCTC