

# Distance and Midpoint Formulas

Guided Notes: TEACHER'S EDITION (DELETE RED FOR STUDENT NOTES)

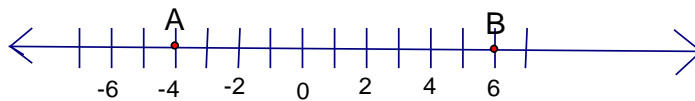
## Question:

How do you determine the distance between two points on a coordinate plane?

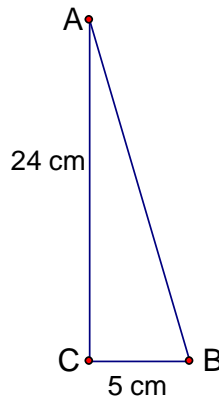
How do you determine the midpoint between two points on a coordinate plane?

## Launch:

- Find the distance between A and B on the number line below. Explain at least two ways you could find this distance. **Try to find more than two. In counting, watch for students who count tick marks instead of spaces.  $|A|+|B|$  or  $|B-A|$**



- Find the midpoint of  $\overline{AB}$  on the number line above. Explain at least two ways you could find the midpoint. **Again emphasize at least two ways. Make connection with average.**



- Find the length of  $\overline{AB}$  in the right triangle above. **Pythagorean Theorem is covered in middle school. Discuss difference between exact answer and approximation.**

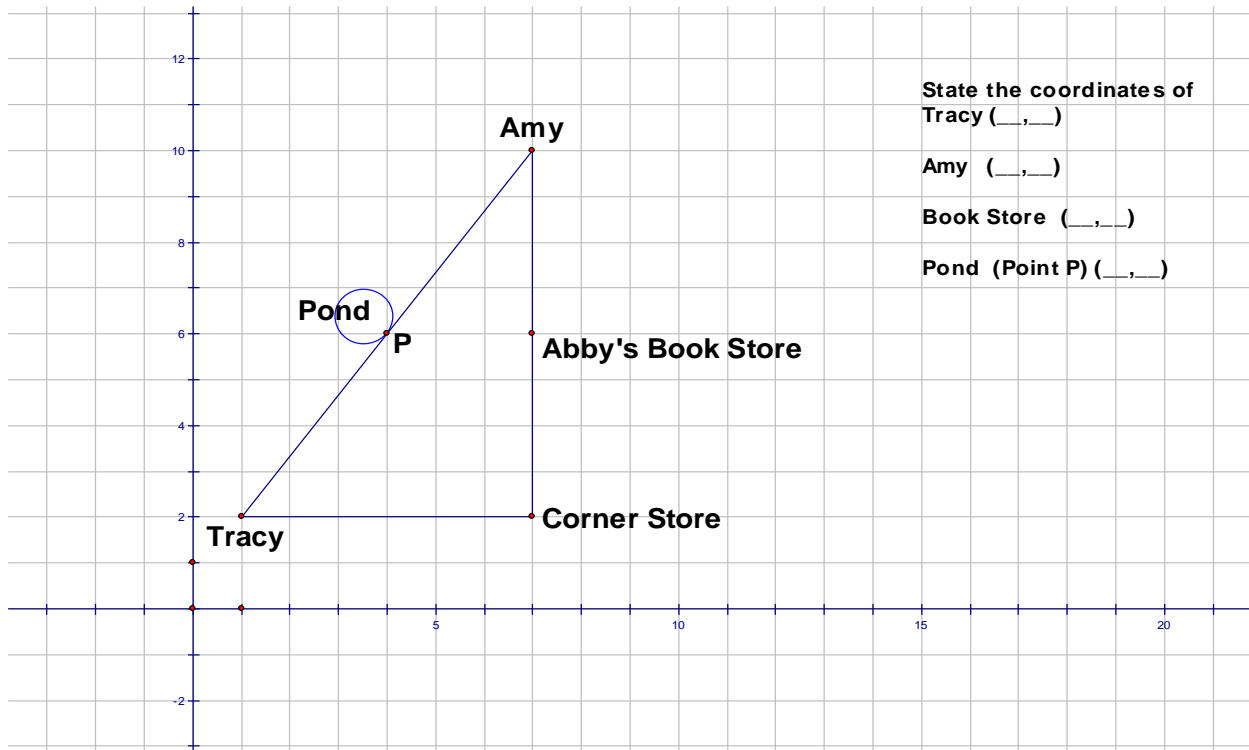
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## Investigation:

Tracy wants to visit Amy for her birthday. She decides to walk to the Corner Store and then pass Abby's Book Store on the way in order to purchase a present. Coming home she will take the shortcut through the park and past the pond.

1. If each unit on the grid represents one block, how many blocks will she walk going to Amy's? How many blocks will she walk going home?



State the coordinates of  
 Tracy (\_\_, \_\_)  
 Amy (\_\_, \_\_)  
 Book Store (\_\_, \_\_)  
 Pond (Point P) (\_\_, \_\_)

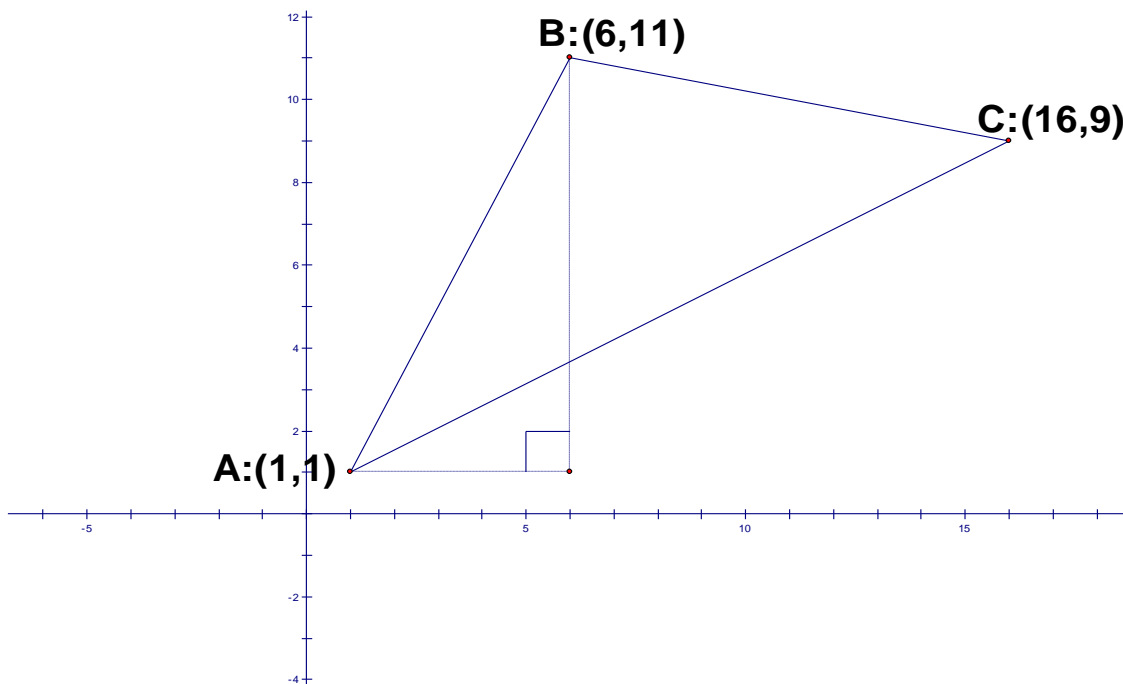
2. Explain how you computed the distance for both trips – coming and going. Give at least two ways of computing the distances.

3. As Tracy is walking home through the field, she stops to dangle her feet in the pond that is exactly half way between Amy and Tracy's house. Give the coordinates of the pond. Explain how you found these coordinates. **Multiple explanations.**

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4. A surveyor must determine the distance around a triangle formed by three towns. He put a grid over the map to help him determine these measurements as shown below. The axes are labeled in miles. Can you help the surveyor find the distance between each town? Maybe the dotted lines that form a right triangle will help you. Explain how you found these distances. **Teacher could encourage them to draw more right triangles with dotted lines.**

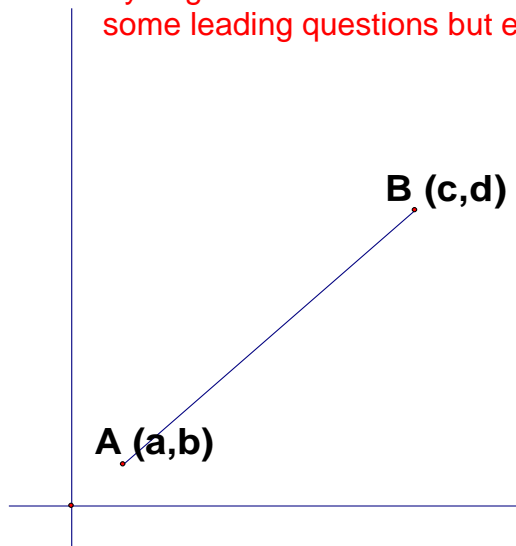


5. A shopping center is to be built midway between Towns A and C. Use the coordinates of A and C to find the coordinates for the shopping center. Explain how you found the mid-point.

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6. Generalize how to find the distance between A and B below? **Students should use the Pythagorean Theorem to develop the Distance Formula. May need to ask some leading questions but encourage students to develop this on their own.**



Look back at how you found the coordinates of the pond in the first problem. Can you generalize this procedure and find the mid-point of segment  $\overline{AB}$  above? **Students should end with a formula for mid-point and distance.**

## Conclusions:

Write a formula (or procedure) to determine the distance between points  $(x_1, y_1)$  and  $(x_2, y_2)$ . **Students should be able to use their own procedure as long as they can explain it carefully.**  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Write a formula to determine the midpoint of the segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$ . **Students should be able to use their own procedure as long as they can explain it carefully.**  $M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

## In Class Problems:

- Find the distance between  $(-8, 7)$  and  $(12, -9)$ .
- Find the midpoint of the segment with endpoints  $(-8, 7)$  and  $(12, -9)$ .
- Three vertices of a triangle are  $X(2, 2)$ ,  $Y(5, 6)$  and  $Z(7, 2)$ . Is the triangle equilateral, scalene, or isosceles? **Students may need a quick reminder here.**

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- The point  $(9, -4)$  is the midpoint of a segment. One endpoint of the segment is located at  $(-8, 2)$ . Find the other endpoint.

## Closure: (Explain in full sentences.)

How do you determine the distance between two points on a coordinate plane?

How do you determine the midpoint between two points on a coordinate plane?

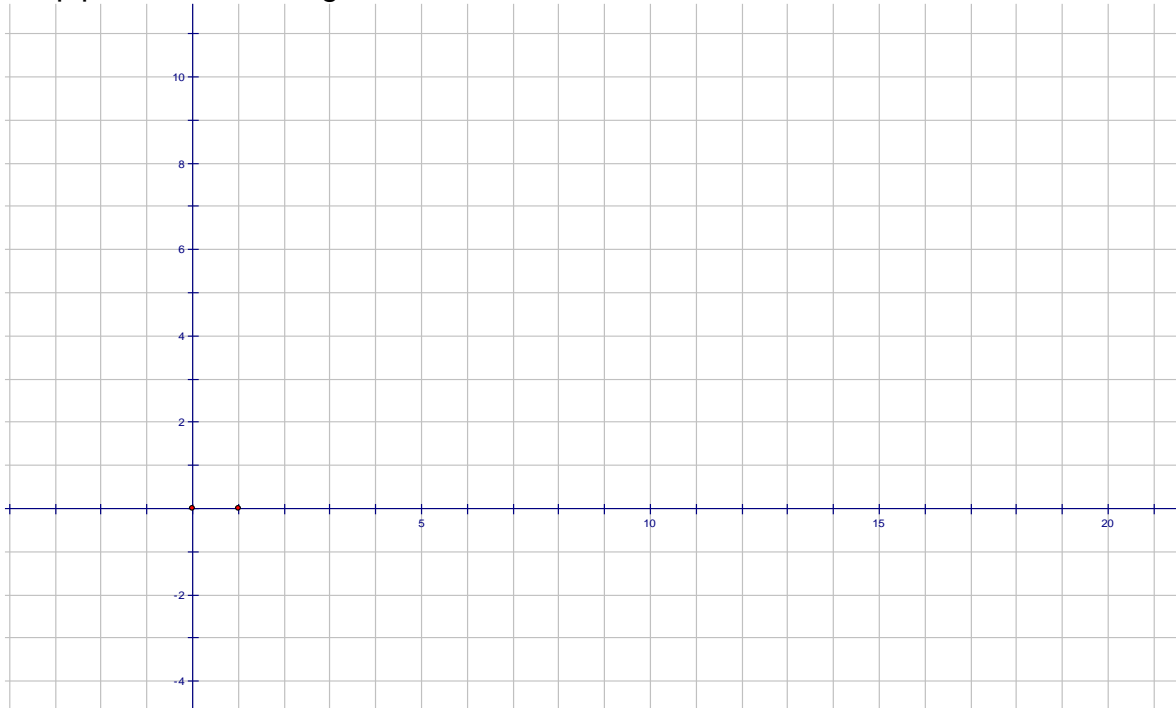
## Homework:

- Find the distance between the given points: **Teacher could add one or two involving fractions or decimals.**
  - $(6, 20)$  and  $(0, -8)$
  - $(-1, -1)$  and  $(-9, -11)$
- A surveyor locates the water well at  $(2, -3)$ . The farmer need to pipe this water to the cattle pond at  $(10.4, 11.2)$ . Sketch these two locations and find the length of

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the pipe. Assume the grid is marked in miles.



- Find the midpoint of the segments with the given endpoints:
  - $(4, -12)$  and  $(-6, 4)$
  - $(18, 28)$  and  $(-7, 15)$
- Find the other endpoint of a line segment with the given midpoint and endpoint:
  - midpoint  $(6, -1)$ ; endpoint  $(-9, 0)$
  - midpoint  $(-8, -8)$ ; endpoint  $(12, 13)$
- How is the distance formula related to the Pythagorean Theorem?
- Prove/verify that the midpoint of the hypotenuse of a right triangle is equidistant from each of the three vertices. **Problem 5 and 6 could wait till next day and teacher could give examples if they choose. These should also lead to a discussion of "prove," "verify," "counterexample," etc.**

Name: \_\_\_\_\_ Teacher: \_\_\_\_\_ Date: \_\_\_\_\_

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7. Find the equation of the line that is the perpendicular bisector of the segment with endpoints  $(-4, 4)$  and  $(6, 2)$ . **Students have had linear functions in grade 8. Thus they should be able to do this. Teacher may need to do a review of vocabulary.**