PLANNING A PROOF

UNIT 2 - LESSON 6

PLANNING A PROOF

Students will be able to:

Understand how to plan the proof of a theorem using the two-columns proof method.

Key Vocabulary

- Theorem, Proof
- Two-columns proof

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What is a theorem?

A **theorem** is a statement that has been proved on the basis of other known theorems and generally accepted statements known as **axioms**.

- Theorem is a logical consequence of axioms.
- Axioms are generally accepted statements (mathematical).
- Axioms are statements that are accepted without proofs.

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What is a proof of a theorem?

A **proof** is a set of logical arguments used to establish the truth of a theorem.

- Each successive step in a theorem is supported by the previous steps.
- Each step in a theorem is based on some property, postulate or definition in mathematics.
- There are two types of proofs namely **direct** and **indirect** proofs.

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- In **direct** proofs, the given statements and the supporting postulates and axioms are used to prove the statement which is to be proved i.e. the statement to be proved is considered true.

- In **indirect** proofs, the statement to be proved is assumed false and a contradictory result is shown indicating that the assumption that the given statement true is false, is not correct. This means that the given statement must be true.

- We will stick to direct proofs in this topic.

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There are three ways of proving a theorem. The most common one is a **Two-column proof.**

Statements	Reasons
Statement 1	Reason 1
Statement 2	Reason 2
⋮	⋮
Statement N	Reason N

The two column-proof has two columns. The first column contains the statements and second column contains the reasons for those statements.

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The important step in using this method is the way it is planned. There are pre-defined steps being followed while writing a two-column proof.

Step 1: Write statement of the theorem to be proved

Step 2: Draw a diagram (if applicable), marked with the given information

Step 3: Re-write the “given” statement(s)

Step 4: Write supporting reasons for each statement in the proof

Step 5: Write the last statement as the “proved” statement

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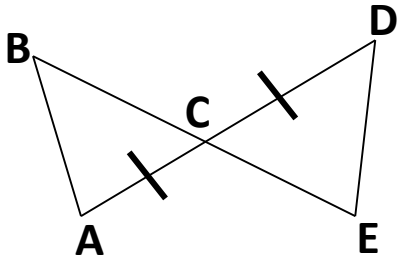
Example 1: Given C is the midpoint of BE and AC ≅ CD, proved that:

$$\Delta ABC \cong \Delta CDE$$



To start the proof, we think about the 5 steps we just went through.

Step 1 and **Step 2** are already being given to us i.e. the statement to be proved and a figure indicating the given.

Step 3 is where the proof will start from. We are given that AC ≅ CD.

Step 4 will contain the statements supported with reasons which we will use to prove the statement (in the next slide)

Step 5 will be statement that has been proved i.e. $\Delta ABC \cong \Delta CDE$

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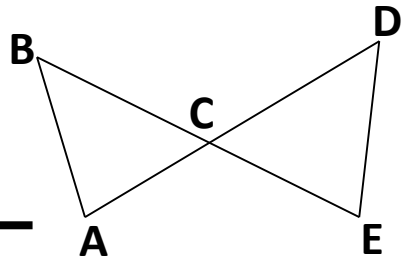
Given C is the midpoint of BE and AC ≅ CD, proved that:

$$\Delta ABC \cong \Delta CDE$$



Proof:

Statements	Reasons
1. AC ≅ CD	1. Given
2. ∠BCA ≅ ∠DCE	2. Vertical angles are congruent
3. C is at the midpoint of BE	3. Given
4. BC ≅ CE	4. Midpoint C divides the segment BE into two congruent segments.
5. ΔABC ≅ ΔCDE	5. SAS postulate.