

The Polygon Angle Sum Theorem Guided Notes

Interior Angles of Polygons

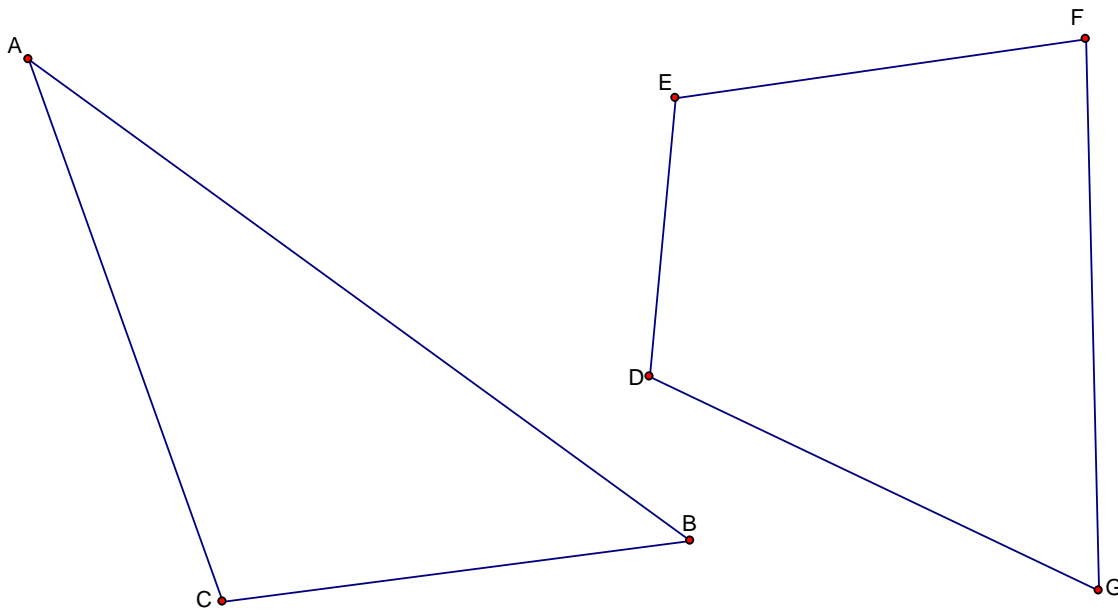
Question:

How can the sum of the interior angles of a polygon be found?

How can the measure of one interior angle of a regular polygon be found?

Launch

With a protractor measure each of the interior angles of Triangle ABC and Quadrilateral DEFG.



1. What is the sum of the interior angles in Triangle ABC?
2. Is the sum of the interior angles of a triangle always the same value? How do you know?
3. What is the sum of the interior angles in Quadrilateral DEFG?
4. Is the sum of the interior angles of a quadrilateral always the same value? How do you know?
5. Can you explain how to find the sum of the interior angles of a triangle or quadrilateral in a different way?

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Investigation:

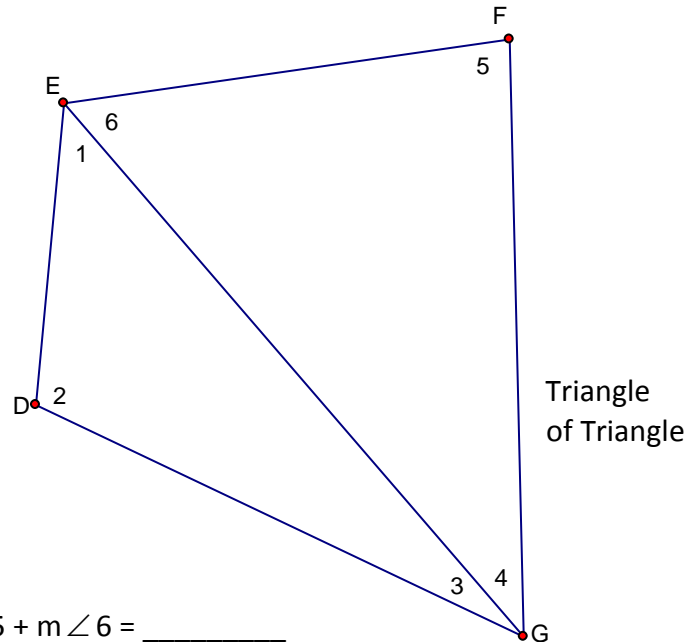
Part 1

In this investigation you are going to discover an easier way to find the sum of the interior angles of a polygon, by dividing a polygon into triangles.

1. Consider the quadrilateral to the right. Diagonal \overline{EG} is drawn. A diagonal is a segment connecting a vertex with a nonadjacent vertex.

The quadrilateral is now divided into two triangles, Triangle DEG and Triangle FEG.

Angles 1, 2, and 3 represent the interior angles of DEG and Angles 4, 5, and 6 represent the interior angles of FEG.



$$3 = \underline{\hspace{2cm}} \angle 2 + m\angle 1 + m\angle 3 \qquad m\angle 4 + m\angle 5 + m\angle 6 = \underline{\hspace{2cm}}$$

2. Mark the interior angles of the triangles with a small arc.

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = \underline{\hspace{2cm}}$$

3. What is the relationship between the sum of the angles in the quadrilateral and the sum of the angles in the two triangles?


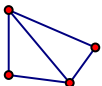
This procedure can be used to find the sum of the interior angles of any polygon.

1. Sketch the polygon.
2. Select one vertex.
3. Draw all possible diagonals from that vertex.
4. Determine the number of triangles formed.

When you know the number of triangles formed, how do you find the sum of the interior angles of the triangle?

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4. Draw a sketch of each polygon and use this same procedure to determine the sum of the angles for each polygon in the table.

Polygon	Sketch	Number of sides	Number of diagonals from 1 vertex	Number of triangles	Interior angle sum
Triangle		3	0	1	180°
Quadrilateral		4	1	2	360°
Pentagon					
Hexagon					
Heptagon					
Octagon					
Decagon					
Dodecagon					
n-gon					

5. What patterns do you notice in the table?

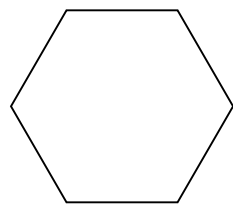
6. In the last row of the table you should have developed a formula for finding the sum of the interior angles of a polygon. Use this formula to find the sum of the interior angles of a 20-gon.

7. Write a sentence explaining how to find the sum of the interior angles of a polygon.

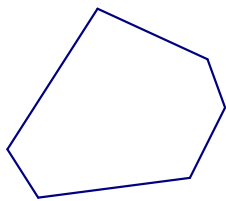
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Part 2



Regular Polygon



Nonregular Polygon

1. Compare the two polygons shown above. How would you define a regular polygon and a nonregular polygon?
2. What is the sum of the interior angles of a hexagon?
3. What is the measure of one angle of a regular hexagon?
4. If you know the sum of the angles of a regular polygon, how can you find the measure of one of the congruent angles?
5. Use the information from Part 1 to complete the table below:

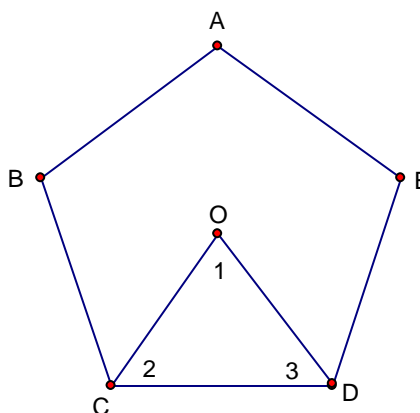
Regular Polygon	Interior angle sum	Measure of one angle
Triangle	180°	
Quadrilateral	360°	
Pentagon		
Hexagon		
Heptagon		
Octagon		
Decagon		
Dodecagon		
n-gon		

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5. Find the measure of one angle of a regular 18-gon.

In Part 1 of this investigation you learned how to find the sum of the interior angles of a polygon by dividing the polygon into triangles. Although this method can be used on any polygon, there is another method that can be used with regular polygons.

The figure to the right is a regular pentagon with center O. Angle COD ($\angle 1$) is called a central angle.



6. How many central angles does a pentagon have?

7. Sketch the other central angles by connecting point O to each vertex of the polygon.

8. If the polygon is regular what conjecture can you make about the lengths of the segments joining point O with each vertex? How do you know?

9. If the polygon is regular what conjecture can you make about the size of the triangles formed?

10. Since all the central angles have a common vertex (Point O), how could you find the measure of one central angle?

11. Why won't this method work on a non-regular polygon?

12. Generalize this by writing a formula that would determine the measure of one central angle in an n-gon.

13. $m\angle COD$ (Angle 1) = _____

14. Triangle COD is an isosceles triangle with vertex angle 1. Why?

15. Find: $m\angle 2$ = _____ $m\angle 3$ = _____

16. What is the measure of $\angle BCO$?

17. What is the measure of $\angle BCD$ (an interior angle of pentagon ABCDE)?

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18. What is the interior angle sum for pentagon ABCDE?
19. Use this method to find the interior angle sum for a regular 30-gon.

Conclusions:

Write a statement explaining how to find each of the following:

1. The sum of the interior angles of a polygon.
2. The measure of one interior angle of a regular polygon.
3. The measure of one central angle of a regular polygon.

In Class Problems:

1. What is another name for a regular triangle?
2. What is another name for a regular quadrilateral?
3. What is the interior angle sum of a 60-gon?
4. What is the measure of one interior angle of a regular 60-gon?
5. Three angles of a quadrilateral measure 98° , 75° , 108° . Find the measure of the fourth angle.
6. Find the measure of each central angle for a regular 20-gon.
7. Each interior angle of a regular polygon measures 168° . How many sides does the polygon have?

Closure:

How can the sum of the interior angles of a polygon be found?

How can the measure of one interior angle of a regular polygon be found?