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Ratios and Proportions

Unit 7 Lesson 1

RATIOS AND PROPORTIONS

Students will be able to:

write and simplify ratios, and
use proportions to solve problems.

Key Vocabulary:

- Ratio
- Extremes
- Cross Product
- Proportion
- Means

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RATIOS AND PROPORTIONS

RATIO is a comparison of a number $\boldsymbol{a}$ and $\boldsymbol{b}$ using division, where $\boldsymbol{b \neq 0}$. It is usually expressed in simplest form and can be expressed as:

$\boldsymbol{a}\text{ to }\boldsymbol{b}$

$\boldsymbol{a}:\boldsymbol{b}$

$\dfrac{\boldsymbol{a}}{\boldsymbol{b}}$

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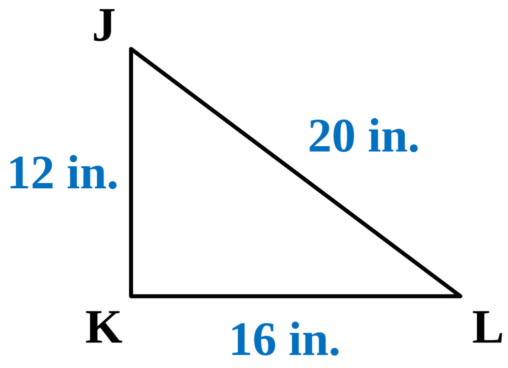
RATIOS AND PROPORTIONS

EXTENDED RATIOS are ratios that can be used to compare three or more quantities.

$$a : b : c \qquad a : b : c : d$$

Example: Express ratio of sides of the triangle in simplest form.



$$\overline{JK} : \overline{KL} : \overline{JL}$$

$$12\ in. : 16\ in. : 20\ in. \rightarrow \frac{12\ in.}{4} : \frac{16\ in.}{4} : \frac{20\ in.}{4}$$

$$\rightarrow \frac{12\ in.}{4\ in.} : \frac{16\ in.}{4\ in.} : \frac{20\ in.}{4\ in.} \rightarrow 3 : 4 : 5$$

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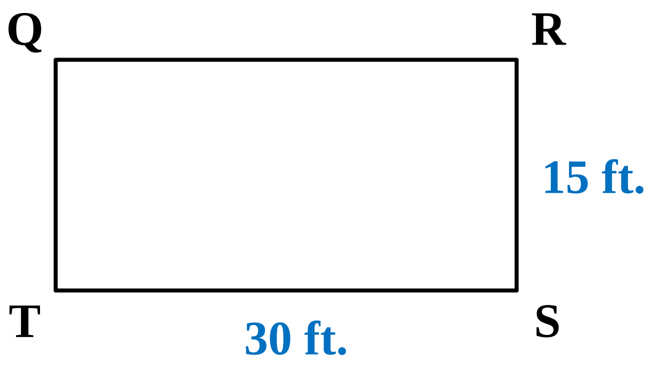
RATIOS AND PROPORTIONS

EQUIVALENT RATIOS are ratios that have the same simplified form.

Example: Express ratio of width and height of the rectangle in simplest form.



$$\frac{width\ of\ the\ rectangle}{height\ of\ the\ rectangle} = \frac{\mathbf{30}\ \boldsymbol{ft}.}{\mathbf{15}\ \boldsymbol{ft}.} = \frac{\mathbf{2}}{\mathbf{1}}$$

Sample Problem 1: Simplify.

a. $\mathbf{35 \text{ to } 7}$

b. $\mathbf{45:63}$

c. $\mathbf{\frac{39}{13}}$

Sample Problem 1: Simplify.

a. $35 \text{ to } 7$

$\rightarrow \quad \frac{35}{7} \text{ to } \frac{7}{7}$

$\rightarrow \quad 5 \text{ to } 1$

b. $45 : 63$

$\rightarrow \quad \frac{45}{9} : \frac{63}{9}$

$\rightarrow \quad 5 : 7$

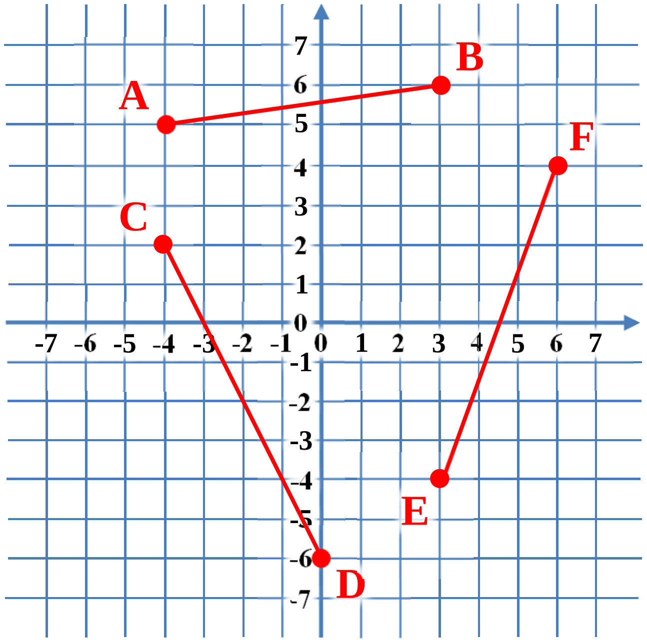
c. $\frac{39}{13}$

$\rightarrow \quad \frac{39}{13}$

$\rightarrow \quad \frac{13(3)}{13(1)}$

$\rightarrow \quad \frac{3}{1}$

Sample Problem 2: Write the ratio expressing the slope of each line segment.

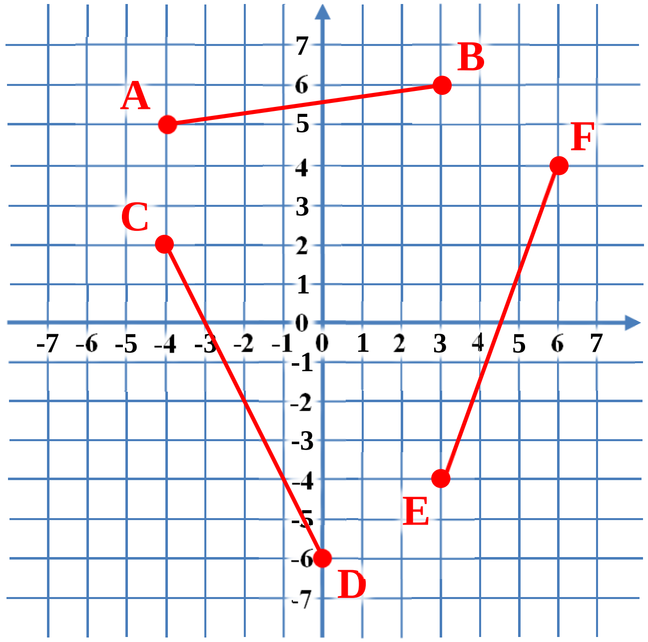


a. $m_{\overline{AB}}$

b. $m_{\overline{CD}}$

c. $m_{\overline{EF}}$

Sample Problem 2: Write the ratio expressing the slope of each line segment.



a. $m_{\overline{AB}} = \frac{y_B - y_A}{x_B - x_A} = \frac{6-5}{3-(-4)} = m_{\overline{AB}} = \frac{1}{7}$

$A(-4, 5)\ B(3, 6)$

b. $m_{\overline{CD}} = \frac{y_D - y_C}{x_D - x_C} = \frac{-6-2}{0-(-4)} = \frac{-8}{4} = m_{\overline{CD}} = -2$

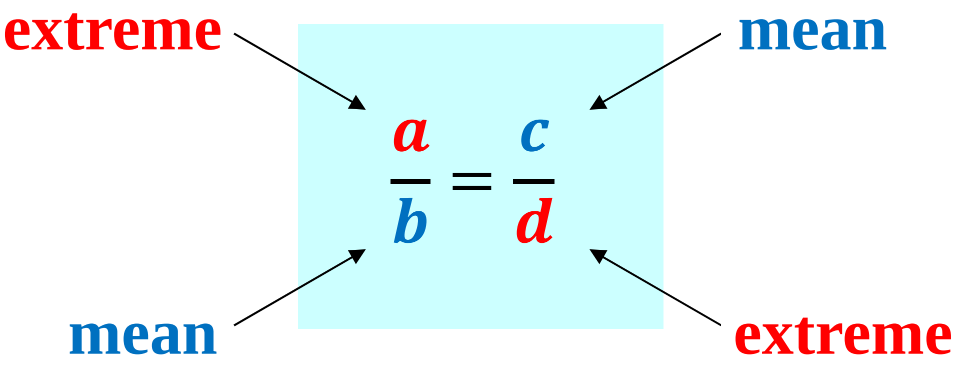
$C(-4, 2)\ D(0, -6)$

c. $m_{\overline{EF}} = \frac{y_F - y_E}{x_F - x_E} = \frac{4-(-4)}{6-3} = m_{\overline{EF}} = \frac{8}{3}$

$E(3, -4)\ F(6, 4)$

RATIOS AND PROPORTIONS

PROPORTION is an equation stating that two ratios are equal.



extreme

mean

$$\frac{a}{b} = \frac{c}{d}$$

mean

extreme

Extremes are the first and last positions in the proportion.

Means are the two middle positions in the proportion.

Sample Problem 3: Solve each proportion.

a. $\frac{x}{11} = \frac{8}{-16}$

b. $\frac{-6}{9} = \frac{8}{2y+7}$

Sample Problem 3: Solve each proportion.

a. $\frac{x}{11} = \frac{8}{-16}$

$\frac{x}{11} = \frac{1}{-2}$

$-2x = 1(11)$

$\frac{-2x}{-2} = \frac{88}{-2}$

$x = -44$

b. $\frac{-6}{9} = \frac{8}{2y+7}$

$-6(2y+7) = 8(9)$

$-12y - 42 = 72$

$-12y - 42 + 42 = 72 + 42$

$-12y = 114$

$\frac{-12y}{-12} = \frac{114}{-12}$

$y = -\frac{19}{2}$

Sample Problem 3: Solve each proportion.

c. $\frac{n+4}{3} = \frac{5n}{6}$

d. $\frac{z+2}{11} = \frac{z-2}{15}$

Sample Problem 3: Solve each proportion.

c. $\frac{n+4}{3} = \frac{5n}{6}$

$6(n+4) \ = 3(5n)$

$6n + 24 = 15n$

$6n - 6n + 24 = 15n - 6n$

$24 = 9n \quad \rightarrow \quad \frac{24}{9} = \frac{9n}{9}$

$\frac{8}{3} = n$

d. $\frac{z+2}{11} = \frac{z-2}{15}$

$\frac{z+1}{11} = \frac{z-2}{15} \quad \rightarrow \quad 15(z+2) = 11(z-2)$

$15z + 30 = 11z - 22$

$15z - 11z + 30 = 11z - 11z - 22$

$4z + 30 = -22$

$4z + 30 - 30 = -22 - 30$

$4z = -52 \quad \rightarrow \quad \frac{4z}{4} = \frac{-52}{4}$

$z = -13$

PROPERTIES OF PROPORTIONS

A. Cross Products Property

In a proportion, the product of the extremes is equal to the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then $= bc$.($b \neq 0$ and $d \neq 0$)

B. Reciprocal Property

If two ratios are equal, then their reciprocals are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.($a \neq 0$ and $c \neq 0$)

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PROPERTIES OF PROPORTIONS

C. If you interchange the means of a proportion, then you form another true proportion.

If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$.($c \neq 0$ and $d \neq 0$)

D. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$. ($b \neq 0$ and $d \neq 0$)

RATIOS AND PROPORTIONS

Sample Problem 4: Solve each proportion.

a. $\frac{3}{7} = \frac{x}{49}$

b. $\frac{2n}{25} = \frac{10}{5n}$

Sample Problem 4: Solve each proportion.

a. $\frac{3}{7} = \frac{x}{49}$

$\frac{3(49)}{7} = x$

$\frac{3(7)}{1} = x$

$21 = x$

b. $\frac{2n}{25} = \frac{10}{5n}$

$\frac{2n}{25} = \frac{10}{5n} \rightarrow \frac{2n}{25} = \frac{2}{n}$

$2n(n) = 2(25) \rightarrow 2n^2 = 50$

$\frac{2n^2}{2} = \frac{50}{2} \rightarrow n^2 = 25$

$n^2 = 25$

$n = \pm 5$

Sample Problem 4: Solve each proportion.

c. $\frac{s}{8}=\frac{16}{2}$

d. $\frac{y+2}{4}=\frac{16}{y+2}$

Sample Problem 4: Solve each proportion.

c. $\frac{s}{8} = \frac{16}{2}$

$$\frac{s}{8} = \frac{16}{2} \rightarrow \frac{s}{8} = \frac{8}{1}$$

$$s = \frac{8(8)}{1} \rightarrow \boxed{s = 64}$$

d. $\frac{y+2}{4} = \frac{16}{y+2}$

$$\frac{y+2}{4} = \frac{16}{y+2}$$

$$(y+2)^2 = 16(4)$$

$$(y+2)^2 = 64 \rightarrow y + 2 = \pm 8$$

$$y + 2 = -8$$

$$y + 2 - 2 = -8 - 2$$

$$\boxed{y = -10}$$

$$y + 2 = 8$$

$$y + 2 - 2 = 8 - 2$$

$$\boxed{y = 6}$$