



**GeometryCoach.com**

# Inequalities in One Triangle

Unit 5 Lesson 6

# INEQUALITIES IN ONE TRIANGLE

**Students will be able to:**

Apply inequalities in one triangle.

**Key Vocabulary:**

- Angle – Side Theorem
- Converse of Angle – side Theorem
- Exterior angle Inequality Theorem
- Triangle Inequality Theorem

# INEQUALITIES IN ONE TRIANGLE

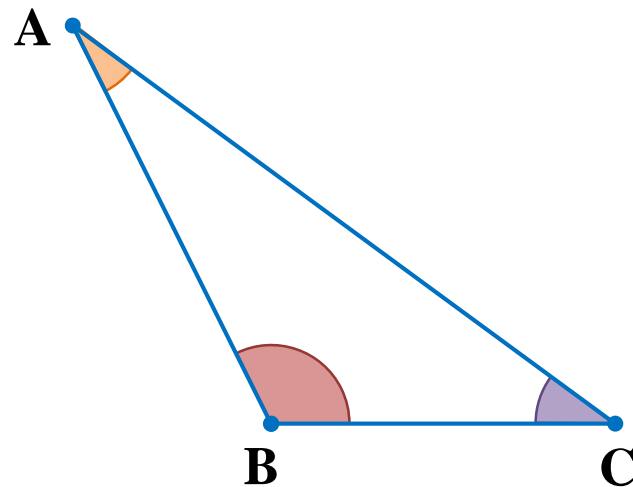
## ANGLE – SIDE THEOREM

“If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.”

If  $\overline{AB} > \overline{BC}$  then  $\angle ACB > \angle BAC$ .

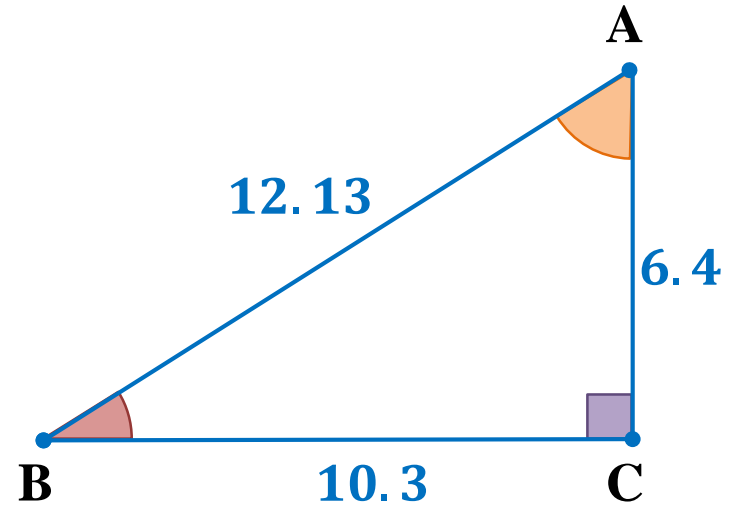
If  $\overline{AC} > \overline{BC}$  then  $\angle ABC > \angle BAC$ .

If  $\overline{AC} > \overline{AB}$  then  $\angle ABC > \angle ACB$ .



# INEQUALITIES IN ONE TRIANGLE

**Sample Problem 1:** Write the angles in order from smallest to largest.



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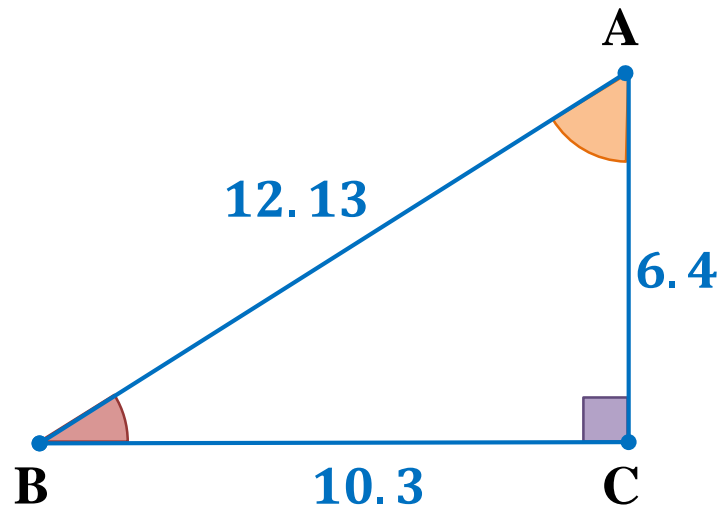
$\overline{AB} = 12.13$  is opposite  $\angle C$

$\overline{BC} = 10.3$  is opposite  $\angle A$

$\overline{AC} = 6.4$  is opposite  $\angle B$

$\overline{AC} < \overline{BC} < \overline{AB}$

$m\angle B < m\angle A < m\angle C$



# INEQUALITIES IN ONE TRIANGLE

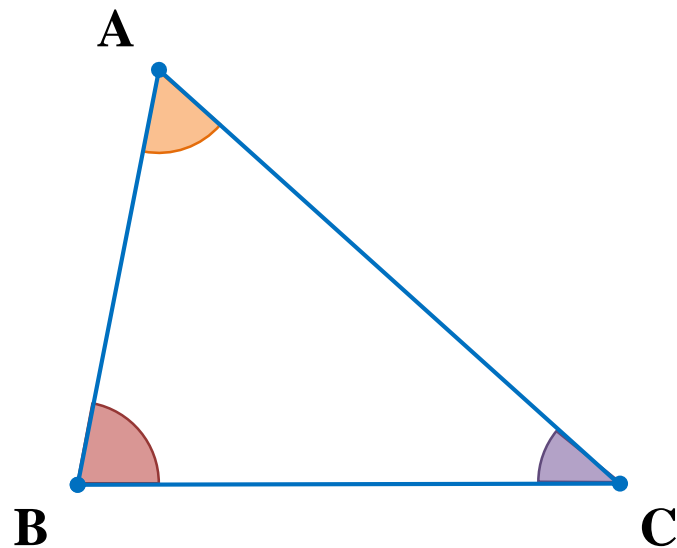
## CONVERSE OF ANGLE – SIDE THEOREM

“If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.”

If  $\angle ACB > \angle BAC$  then  $\overline{AB} > \overline{BC}$ .

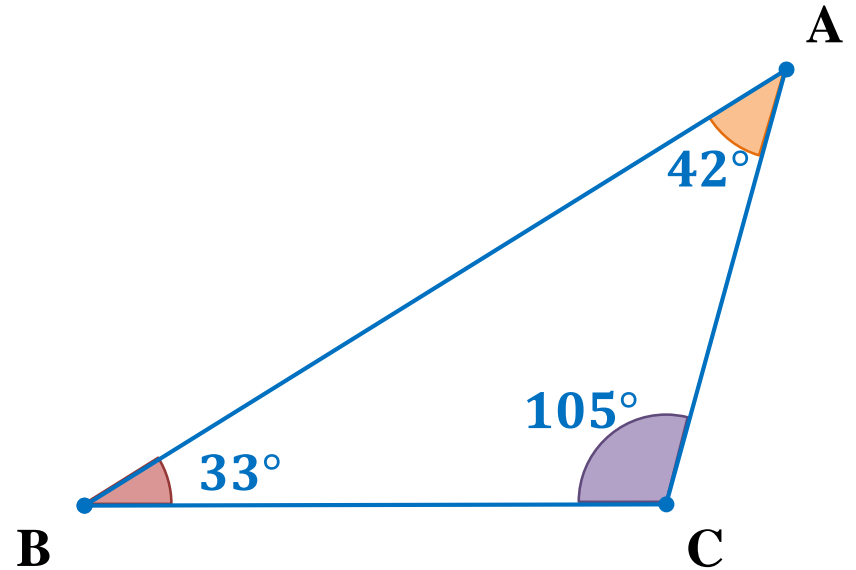
If  $\angle ABC > \angle BAC$  then  $\overline{AC} > \overline{BC}$ .

If  $\angle ABC > \angle ACB$  then  $\overline{AC} > \overline{AB}$ .



# INEQUALITIES IN ONE TRIANGLE

**Sample Problem 2:** Write the sides in order from shortest to longest.



# INEQUALITIES IN ONE TRIANGLE

**Sample Problem 2:** Write the sides in order from shortest to longest.

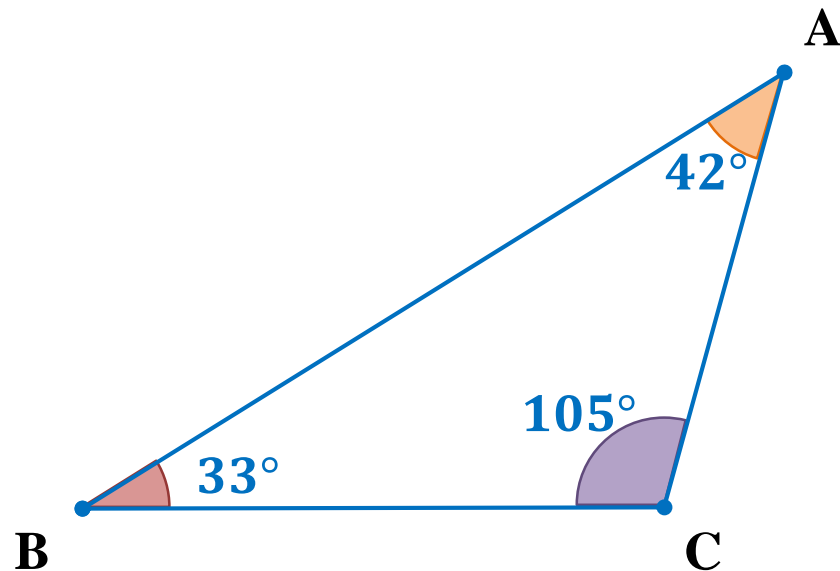
$\overline{AB}$  is opposite  $m\angle C = 105^\circ$

$\overline{BC}$  is opposite  $m\angle A = 42^\circ$

$\overline{AC}$  is opposite  $m\angle B = 33^\circ$

$m\angle B < m\angle A < m\angle C$

$\overline{AC} < \overline{BC} < \overline{AB}$





# INEQUALITIES IN ONE TRIANGLE

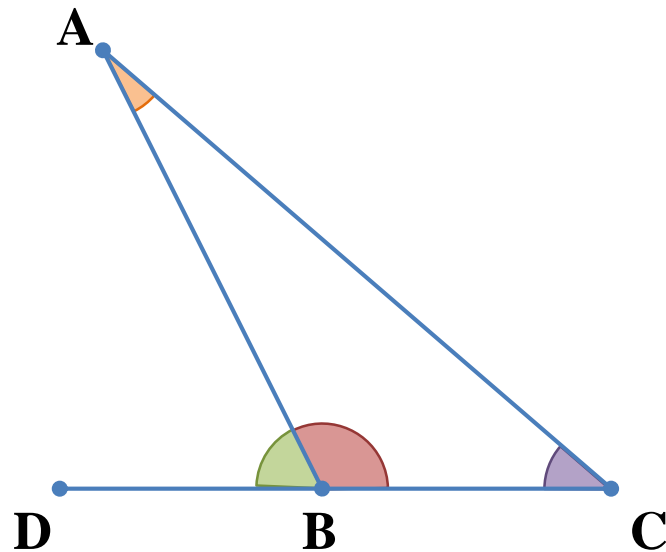
## EXTERIOR ANGLE INEQUALITY THEOREM

“The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.”

$$\angle ABD > \angle BAC$$

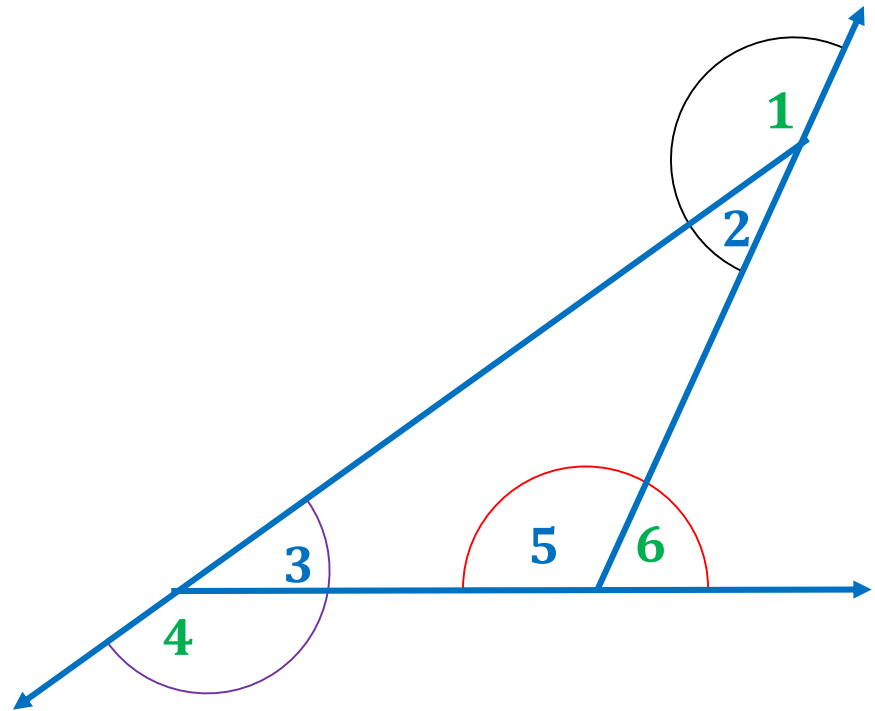
and

$$\angle ABD > \angle BCA$$



# INEQUALITIES IN ONE TRIANGLE

**Sample Problem 3:** Determine the smallest and the largest angles.



# INEQUALITIES IN ONE TRIANGLE

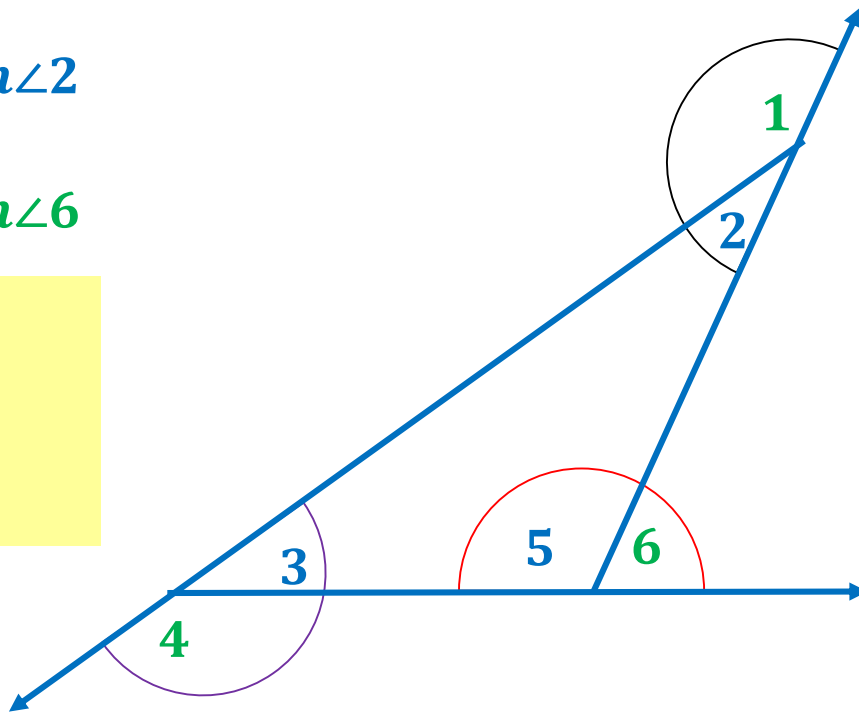
**Sample Problem 3:** Determine the smallest and the largest angles.

Interior angles:  $m\angle 5 > m\angle 3 > m\angle 2$

Exterior angles:  $m\angle 1 > m\angle 4 > m\angle 6$

Smallest angle:  $m\angle 2$

Largest angle:  $m\angle 1$



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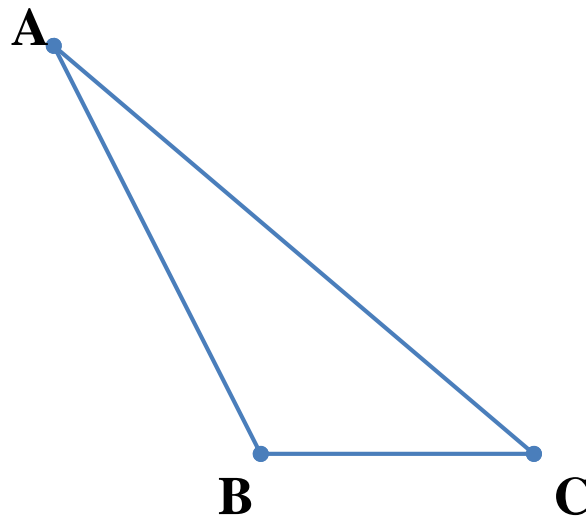
## TRIANGLE INEQUALITY THEOREM

“The sum of the lengths of any two sides of a triangle is greater than the length of the third side.”

$$\overline{AB} + \overline{BC} > \overline{AC}$$

$$\overline{AC} + \overline{BC} > \overline{AB}$$

$$\overline{AB} + \overline{AC} > \overline{BC}$$



## INEQUALITIES IN ONE TRIANGLE

**Sample Problem 4:** A triangle has one side of length 12 and another of length 8. Identify the possible lengths of the third side.

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**Sample Problem 4:** A triangle has one side of length 12 and another of length 8. Identify the possible lengths of the third side.

$$x = 12$$

$$x + y > z$$

$$12 + 8 > z$$

$$20 > z$$

$$y = 8$$

$$x + z > y$$

$$12 + z > 8$$

$$z > 8 - 12$$

$$z > -4$$

$$4 < z < 20$$

$$y + z > x$$

$$8 + z > 12$$

$$z > 12 - 8$$

$$z > 4$$